Reg. No.

Question Paper Code : 51771

B.E/B.Tech. DEGREE EXAMINATION, MAY/JUNE 2016

Second Semester

Civil Engineering

MA 2161/MA 22/080030004 – MATHEMATICS – II

(Common to all Branches)

(Regulations 2008)

Time : Three Hours **Maximum : 100 Marks**

Answer ALL questions.

PART – A (10 × 2 = 20 Marks)

1. Solve $(D^2 + 6D + 9)y = 0$

2. Find the particular integral of $(D^2 + 4D + 8)y = e^{2x}$

3. If $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$, then show that grad $r = \frac{\vec{r}}{r}$

4. Find the divergence of the vector field $\vec{A} = (xyz)\hat{i} + (3x^2y)\hat{j} + (xz^2 - y^2z)\hat{k}$ at the point $(2, -1, 1)$.

5. Define analytic function and give a suitable example.

6. Define harmonic function and give a suitable example.

7. Evaluate $\oint_C (x^2 - y^2 + 2ixy)dz$, where C is the contour $|z| = 1$

8. Expand the function $f(z) = \frac{e^z}{(z-1)^2}$ about z = 1 in a Laurent's series.

9. If c_1 and c_2 are constant and f and g are functions of t then show that

$L\{c_1 f(t) + c_2 g(t)\} = c_1 L\{f(t)\} + c_2 L\{g(t)\}$

10. If $L\{F(t)\} = \frac{1}{p(p+\beta)}$ where 'β' is a constant, then find $\lim_{t \to \infty} F(t)$.

PART – B (5 × 16 = 80 Marks)

11. (a) (i) Solve $(D^2 + 7D + 12)y = 14e^{-3x}$ **(8)**

(ii) Solve $(x^2D^2 + 4xD + 2)y = x \log x$ **(8)**

OR

(b) (i) Solve $(D^2 + 5D + 6)y = 4 \cos 5x$ **(8)**

(ii) Solve $\frac{dx}{dt} + 2x - 3y = t$, $\frac{dy}{dt} - 3x + 2y = e^{2t}$ **(8)**

12. (a) (i) Show that the vector field $\vec{A} = (x^2 + xy^2)\hat{i} + (y^2 + x^2y)\hat{j}$ is irrotational and find the scalar potential. **(8)**

(ii) Using Green's theorem show that the area bounded by a simple closed curve C is given by $\frac{1}{2}\int (xdy - ydx)$. Hence find the area of an ellipse. **(8)**

OR

(b) Verify Gauss Divergence theorem for $\vec{F} = 4x\hat{i} - 2y^2\hat{j} + z^2\hat{k}$ taken over the region bounded by the cylinder $x^2 + y^2 = 4$, z = 0 and z = 3. **(16)**

13. (a) (i) Show that (i) an analytic function with constant real part is a constant and (ii) an analytic function with constant modulus is also a constant. (8)

(ii) If f(z) is a regular function of z then prove that $\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)|f(z)|^2 = 4|f'(z)|^2$. (8)

OR

(b) (i) Show that $v(x, y) = \log(x^2 + y^2)$ is a harmonic function. Find a function $u(x, y)$ such that $f(z) = u(x, y) + iv(x, y)$ is an analytic function. (8)

(ii) Find the image of the circle $|z-1| = 1$ in the complex plane under the mapping $w = \frac{1}{z}$. (8)

14. (a) (i) Evaluate $\oint_C \left(\frac{z^2+1}{z^2-1}\right) dz$ where C is a circle (8)

(1) $|z| = 3/2$

(2) $|z| = 1/2$.

(ii) Expand

(1) $f(z) = \frac{\sin z}{z-\pi}$ about $z = \pi$ and

(2) $f(z) = \frac{z}{(z+1)(z+2)}$ about $z = -2$. (8)

OR

(b) (i) Using contour integration prove that $\int_{-\infty}^{+\infty} \frac{dx}{(1+x^2)^2} = \frac{\pi}{2}$ (8)

(ii) Evaluate $\int_0^{2\pi} \frac{d\theta}{2+\cos\theta}$, using contour integration. (8)

15. (a) (i) Find Laplace transform of $F(t) = (1 + t\,e^{-2t})^3$. (8)

(ii) Find the Inverse Laplace transform of $f(p) = \dfrac{p^2 + 2p - 3}{p(p-3)(p+2)}$ (8)

OR

(b) (i) Find Laplace transform of $F(t) = \dfrac{1 - \cos t}{t^2}$ (8)

(ii) State convolution theorem and hence evaluate $L^{-1}\left\{\dfrac{p}{(p^2+1)(p^2+4)}\right\}$ (8)
