24-5 FN

Reg. No. : [ ][ ][ ][ ][ ][ ][ ][ ][ ][ ][ ][ ]

Question Paper Code : 72065

B.E./B.Tech. DEGREE EXAMINATION, APRIL/MAY 2017.

Second Semester

Civil Engineering

MA 6251 — MATHEMATICS – II

(Common to Mechanical Engineering (Sandwich), Aeronautical Engineering, Agriculture Engineering, Automobile Engineering, Biomedical Engineering, Computer Science and Engineering, Electrical and Electronics Engineering, Electronics and Communication Engineering, Electronics and Instrumentation Engineering, Environmental Engineering, Geoinformatics Engineering, Industrial Engineering, Industrial Engineering and management, Instrumentation and Control Engineering, Manufacturing Engineering, Materials Science and Engineering, Mechanical Engineering, Mechanical and Automation Engineering, Mechatronics Engineering, Medical Electronics Engineering, Petrochemical Engineering, Production Engineering, Robotics and Automation Engineering, Biotechnology, Chemical Engineering, Chemical and Electrochemical Engineering, Fashion Technology, Food Technology, Handloom and Textile technology, Information Technology, Petrochemical Technology, Petroleum Engineering, Pharmaceutical Technology, Plastic Technology, Polymer Technology, Textile Chemistry, Textile Technology, Textile Technology (Fashion Technology))

(Regulations 2013)

Time : Three hours | Maximum : 100 marks

Answer ALL questions.

PART A — ($10 \times 2 = 20$ marks)

1. Find the unit normal vector to the surface $x^3 + y^2 = z$ at $(1,1,2)$.

2. Using Green's theorem in the plane, find the area of the circle $x^2 + y^2 = a^2$.

3. Find the particular integral of the equation $(D^2 + 4D + 4)\,y = e^{-2x}$.

4. Solve : $x^2 \dfrac{d^2y}{dx^2} + 2x\dfrac{dy}{dx} = 0$.

5. State sufficient condition for the existence of Laplace transform.

6. Find the inverse Laplace transform of $\frac{s^2 - 3s + 2}{s^3}$.

7. The real part of an analytic function $f(z)$ is constant, prove that $f(z)$ is a constant function.

8. Find the critical points of the transformation $w = z^2 + \frac{1}{z^2}$.

9. Evaluate $\int_C \frac{e^z dz}{(z-2)}$, where C is the unit circle with centre as origin.

10. Determine the residue of $f(z) = \frac{z+1}{(z-1)(z+2)}$ at $z = 1$.

PART B — (5 × 16 = 80 marks)

11. (a) (i) Find the angle between the surfaces $x^2 + y^2 + z^2 = 9$ and $z = x^2 + y^2 - 3$ at the point $(2, -1, 2)$. (8)

(ii) Verify Stoke's theorem for $\vec{F} = (x^2 - y^2)\vec{i} + 2xy\vec{j}$, where S is the rectangle in the xy-plane formed by the lines $x = 0$, $x = a$, $y = 0$ and $y = b$. (8)

Or

(b) (i) Find the constants a, b, c so that $\vec{F} = (x + 2y + az)\vec{i} + (bx - 3y - z)\vec{j} + (4x + cy + 2z)\vec{k}$ is irrotational. For those values of a, b, c find its scalar potential. (6)

(ii) Verify Divergence theorem for $\vec{F} = 4xz\vec{i} - y^2\vec{j} + yz\vec{k}$ taken over the cube bounded by the planes $x = 0$, $x = a$, $y = 0$, $y = a$, $z = 0$, $z = a$. (10)

12. (a) (i) Solve : $(D^2 + 5D + 4)y = e^{-x}\sin 2x + 2e^{-x}$. (8)

(ii) Solve the differential equation $(D^2 + 4)y = \sec^2 2x$ by the method of variation of parameters. (8)

Or

(b) (i) Solve : $(1+x)^2 \frac{d^2y}{dx^2} + (1+x)\frac{dy}{dx} + y = 2\sin[\log(1+x)]$. (8)

(ii) Solve : $(D+2)x + 3y = 2e^{2t}$; $3x + (D+2)y = 0$. (8)

13. (a) (i) Evaluate : $\int_0^\infty e^{-t}\left(\frac{\cos 2t - \cos 3t}{t}\right) dt$. (8)

(ii) Apply convolution theorem to evaluate $L^{-1}\left\{\frac{s^2}{(s^2+4)\,(s^2+9)}\right\}$. (8)

Or

(b) (i) (1) Find the Laplace transform of $f(t) = t\,e^{-2t}\cos 3t$. (5)

(2) Find $L^{-1}\left\{\log\left(\frac{s^2+4}{(s-2)^2}\right)\right\}$. (5)

(ii) Using Laplace transform, solve the differential equation $\frac{d^2y}{dt^2} + 3\frac{dy}{dt} + 2y = e^{-t}$, $y(0) = 1$, $y'(0) = 0$. (6)

14. (a) (i) If $f(z)$ is a regular function of z, prove that $\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)\log|f(z)| = 0$. (8)

(ii) Show that the transformation $w = \frac{1}{z}$ transforms in general, circles and straight lines into circles and straight lines. (8)

Or

(b) (i) Find the analytic function $f(z) = u + iv$, given that $2u + 3v = e^x(\cos x - \sin y)$. (8)

(ii) Find the bilinear transformation which maps the point –1, 0, 1 of the z-plane into the points $-1, -i, 1$ of the w-plane respectively. (8)

15. (a) (i) Evaluate $\int_C \frac{(z+1)\,dz}{(z-1)(z-2)^2}$, where C is the circle $|z-2| = \frac{1}{2}$ using Cauchy's integral formula. (8)

(ii) Find the Laurent series expansion of $f(z) = \frac{1}{z^2+4z+3}$ valid in the regions $|z| < 1$ and $0 < |z+1| < 2$. (8)

Or

(b) (i) Evaluate $\int_0^{2\pi} \frac{d\theta}{1-2x\cos\theta+x^2}$ $(0 < x < 1)$ using contour integration. (8)

(ii) Evaluate $\int_0^{\infty} \frac{x^2 dx}{(x^2+4)(x^2+9)}$ using contour integration. (8)
