21-4 AN

Reg. No. : [ ]

Question Paper Code : 73767

B.E./B.Tech. DEGREE EXAMINATION, APRIL/MAY 2017.

Second Semester

Civil Engineering

MA 2161/MA 22/080030004 — MATHEMATICS — II

(Common to all Branches)

(Regulations 2008)

Time : Three hours | Maximum : 100 marks

Answer ALL questions.

PART A — (10 × 2 = 20 marks)

1. Solve $(D^3 - 5D^2 + 8D - 4)y = 0$.

2. Find the particular integral of $(D^2 - 5D + 6)y = 2\cosh 2x$.

3. What is the greatest rate of $\phi = xyz^2$ at $(1, 0, 3)$?

4. Find the unit normal vector to the surface $x^2 + y^2 - z = 1$ at $(1,1,1)$.

5. If $u(x, y) + iv(x, y)$ is analytic, then show that $u(x, y) = c_1$ and $v(x, y) = c_2$ are orthogonal.

6. Show that $f(x, y) = \log\sqrt{x^2 + y^2}$ is harmonic.

7. Classify the singularities for the function $f(z) = \dfrac{z - \sin z}{z}$.

8. Evaluate $\int_C \dfrac{e^z}{z-1}\,dz$, where C is $|z+3| = 1$.

9. Find $L\left(\sqrt{t} - \dfrac{1}{\sqrt{t}}\right)$.

10. State initial and final value theorem of Laplace transform.

PART B — (5 × 16 = 80 marks)

11. (a) (i) Solve by the method of variation of parameters $(D^2 + a^2)y = \tan ax$. (8)

(ii) Solve $(x^2D^2 - 3xD + 5)y = x^2 \sin(\log x)$. (8)

Or

(b) (i) Solve $\frac{dx}{dt} + 2y + \sin t = 0$; $\frac{dy}{dt} - 2x - \cos t = 0$. (8)

(ii) Solve $(D^2 - 2D + 4)y = e^x \cos^2 x$. (8)

12. (a) (i) Prove that $\text{div}(\text{grad}\, r^n) = n(n+1)r^{n-2}$. (4)

(ii) Prove that curl (grad ϕ) = 0, using Stoke's theorem. (4)

(iii) Show that $\vec{F} = 2xyz^3\vec{i} + x^2z^3\vec{j} + 3x^2yz^2\vec{k}$ is irrotational. Find the scalar potential ϕ such that $F = \text{grad}\,\phi$. (8)

Or

(b) (i) Verify Gauss divergence theorem for $\vec{F} = y\vec{i} + x\vec{j} + z^2\vec{k}$ for the cylindrical region S given by $x^2 + y^2 = a^2$, $z = 0$, $z = h$. (8)

(ii) Find the work done in a moving a particle in the force field $\vec{F} = 3x^2\vec{i} + (2xz - y)\vec{j} + z\vec{k}$ along the curve defined by $x^2 = 4y$, $3x^2 = 8z$ from $x = 0$ to $x = 2$. (8)

13. (a) (i) If $f(z)$ is a regular function of z, then prove that $\nabla^2|f(z)|^2 = 4|f'(z)|^2$. (8)

(ii) Find an analytic function $f(z) = u + iv$, given that $2u + 3v = \frac{\sin 2x}{\cosh 2y - \cos 2x}$. (8)

Or

(b) (i) Find the image of the infinite strip $\frac{1}{4} < y < \frac{1}{2}$ under the transformation $w = \frac{1}{z}$. (8)

(ii) Find the bilinear transformation which maps the points $z = \infty$, $z = i$, $z = 0$ on to the points $w_1 = 0$, $w_2 = i$, $w_3 = \infty$. (8)

14. (a) (i) By using Cauchy's integral formula, evaluate $\int_C \frac{z dz}{(z-2)(z-3)^2}$ where C is $|z-3| = \frac{1}{2}$. (8)

(ii) By using contour integration, evaluate $\int_0^{2\pi} \frac{d\theta}{2 + \sin\theta}$. (8)

Or

(b) (i) Find the Laurent series expansion of the function $f(z) = \frac{7z-2}{z(z+1)(z+2)}$ in the annular region $1 < |z+1| < 3$. (8)

(ii) By using contour integration, evaluate $\int_{-\infty}^{\infty} \frac{x^2\, dx}{(x^2+a^2)(x^2+b^2)}$. (8)

15. (a) (i) Evaluate $\int_0^{\infty} \frac{e^{\sqrt{2}t} \sin\sqrt{2}\,t}{t}\, dt$. (8)

(ii) Using Laplace transform, solve $y'' - 4y' + 8y = e^t$, given that $y(0) = 2$, $y'(0) = 1$. (8)

Or

(b) (i) Using convolution theorem, find $L^{-1}\left(\frac{s}{(s^2+a^2)^2}\right)$. (8)

(ii) Find the Laplace transform of the function

$$f(t) = \begin{cases} a\sin\omega t, & 0 \le t \le \pi/\omega \\ 0, & \pi/\omega \le t \le 2\pi/\omega \end{cases}$$ and $f(t + 2\pi/\omega) = f(t)$. (8)

———————