ANNA UNIVERSITY COIMBATORE

B.E. / B.TECH. DEGREE EXAMINATIONS : JUNE – JULY 2009

REGULATIONS - 2008

SECOND SEMESTER

080030004 - MATHEMATICS - II

(COMMON TO ALL BRANCHES)

TIME : 3 Hours **Max. Marks : 100**

PART – A

(20 x 2 = 40 MARKS)

ANSWER ALL QUESTIONS

1. Find the particular integral of $(D^2+1)y = \cosh 2x$.
2. Solve $(x^2D^2 + xD)y = 0$.
3. Transform $(1+x)^2\frac{d^2y}{dx^2} + (1+x)\frac{dy}{dx} + y = 2\sin(\log(1+x))$ into linear equation with constant coefficients
4. Eliminate x and find the equation in y from $\frac{dx}{dt}+5x-2y=t,\ \frac{dy}{dt}+2x+y=0$
5. Find the maximum directional derivative of $\Phi = x^2yz + 4xz^2$ at the point $P(1,-2,-1)$.
6. Show that $\overline{F} = (y^2 - z^2 + 3yz - 2x)\overline{i} + (3xz + 2xy)\overline{j} + (3xy - 2xz + 2z)\overline{k}$ is solenoidal
7. State Green's theorem in a plane.
8. If $\overline{F}$ is irrotational and C is closed curve then find the value of $\int_C \overline{F}\,dr$.
9. State C – R equations in Cartesian coordinates.
10. Prove that $u = 3x^2y + 2x^2 - y^3 - 2y^2$ is a harmonic function.

11. Find the image of the circle $|z-\alpha| = r$ by the mapping $W = Z + C$, where c is a constant.
12. Find the fixed points of the mapping $w = \frac{1}{z+2i}$
13. State Cauchy's Integral formula.
14. Evaluate $\int_C \frac{3z^2+7z+1}{(z-3)}dz$, where C is $|z| = 2$.
15. Find the Laurent's series of $f(z) = z^2e^{1/z}$ about $z = 0$.
16. Find the residue of $f(z) = \frac{z^2}{(z-1)^2(z+2)}$ at $z=-2$.
17. State the sufficient conditions for the existence of Laplace transform of $f(t)$
18. If $L[f(t)] = F(s)$, prove that $L[f(at)] = \frac{1}{a}F\left(\frac{s}{a}\right)$
19. Find $L(e^{-at}\sin bt)$
20. Find $L^{-1}\left[\frac{1}{(s+2)^3}\right]$

PART – B

(5 x 12 = 60 MARKS)

ANSWER ANY FIVE QUESTIONS

21. a) Solve: $\frac{dx}{dt} + 2y = \sin 2t,\ \frac{dy}{dt} - 2x = \cos 2t$ 6

    b) Solve $(x^2D^2 - 7xD + 12)y = x$ 6

22. Verify Gauss divergence theorem for $\overline{F} = x^2\overline{i} + y^2\overline{j} + z^2\overline{k}$, where S is the surface of the cuboid formed by the planes x = 0, x = 1, y = 0, y = 2, z = 0, z = 3. 12

23. a) Prove that $x^2 - y^2 + e^{-2x}\cos 2y$ is harmonic and find its harmonic conjugate. 6

b) Find the bilinear transformation which maps the points $z = -1, 0, i$ into $w = -1, i, 1$ respectively. 6

24. a) Using Cauchy's integral formula evaluate $\int_c \frac{\sin \pi z^2 + \cos \pi z^2}{(z-1)(z-2)} dz, C\ is\ |z| = 3.$ 6

b) Using residue calculus prove that $\int_0^{2\pi} \frac{d\theta}{5 + 3\cos\theta} = \frac{\pi}{2}.$ 6

25. a) Find the Laplace transform of the periodic function f(t) defined by 8

$$f(t) = \begin{Bmatrix} k; 0 \le t \le a \\ -k; a \le t \le 2a \end{Bmatrix} \& f(t + 2a) = f(t)$$

b) Find the Inverse Laplace Transform of $\frac{s-3}{s^2 + 4s + 13}$ 4

26. a) Using Cauchy's residue theorem evaluate $\int_C \frac{z-1}{(z+1)^2 (z-2)} dz$, C is $|z-i| = 2$. 6

b) Prove that $\vec{F} = (y^2 \cos x + z^3)\vec{i} + (2y\sin x - 4)\vec{j} + 3xz^2\vec{k}$ is irrotational and find its scalar potential. 6

27. a) Find the image of the infinte strip $\frac{1}{4} \le y \le \frac{1}{2}$ under the transformation $w = \frac{1}{z}$ 6

b) Solve: $(D^2 - 2D + 5)y = e^{2x}\sin x$ 6

28. Using Laplace Transform solve 12

$$(D^2 + 4D + 13) y = e^{-t}\sin t, y(0) = 0 \& Dy = 0\ at\ t = 0\ where\ D = \frac{d}{dt}$$

*****THE END*****