ANNA UNIVERSITY OF TECHNOLOGY, COIMBATORE

B.E. / B.TECH. DEGREE EXAMINATIONS : NOV / DEC 2011

REGULATIONS : 2008

SECOND SEMESTER

080030004 - MATHEMATICS - II

(COMMON TO ALL BRANCHES)

Time : 3 Hours | Max.Marks : 100

PART - A

(10 x 2 = 20 MARKS)

ANSWER ALL QUESTIONS

1. Find the particular integral of $(D^2+2D+5)y = e^x \cos 3x$.
2. Solve: $((x+1)^2 D^2 + (x+1)D + 1)y = 0$.
3. Show that $\vec{F} = (y^2 - 2xz^2)\vec{i} + (2xy - z)\vec{j} + (2x^2 z - y + 2z)\vec{k}$ is irrotational.
4. State the Stokes' theorem.
5. Write the necessary condition for the function f(z) should be analytic.
6. Find the image of the circle $|z| = 2$ by the transformation $w = z + 3 + 2i$.
7. State Cauchy Integral Theorem.
8. Find the residue of $f(z) = \frac{1}{(z^2+a^2)^2}$ at z=ai.
9. Find $L\left[\frac{1-e^t}{t}\right]$.
10. Find $L^{-1}\left[\frac{s}{(s+2)^2+1}\right]$.

PART - B

(5 x 16 = 80 MARKS)

ANSWER ALL QUESTIONS

11. (a) (i) Solve $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = xe^x \sin x$. 8

(ii) Solve: $(x^2D^2 - 4xD + 2)y = x\log x$. 8

(OR)

11. (b) (i) Solve $(D^2+2D+5)y = e^{-x}\tan x$ by using method of variation of parameters. 8

(ii) Solve the simultaneous equations: $\frac{dx}{dt} + 2x - 3y = t,\ \frac{dy}{dt} - 3x + 2y = e^{2t}$ 8

12. (a) Verify divergence theorem for $\vec{F} = 4x\vec{i} - 2y^2\vec{j} + z^2\vec{k}$ taken over the region bounded by $x^2 + y^2 = 4, z = 0$ and $z = 3$

(OR)

12. (b) (i) Find the work done by the force $\vec{F} = (2xy + z^3)\vec{i} + x^2\vec{j} + 3xz^2\vec{k}$ when it moves a particle from $(1,-2,1)$ to $(3,1,4)$ along any path. 8

(ii) By applying Green's theorem that the area bounded by a simple closed curve C is $\frac{1}{2}\int_C (xdy - ydx)$ and hence find thae area of the ellipse. 8

13. (a) (i) Show that $\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)=4\frac{\partial^2}{\partial z\,\partial \bar{z}}$ and also if $f(z)=u+iv$ is analytic, prove that $\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)\log|f'(z)|=0$. 8

(ii) Show that an analytic function with constant modulus is constant. 8

(OR)

13. (b) (i) If $f(z)=u+iv$ is an analytic function and $u-v=e^x(\cos y-\sin y)$ find $f(z)$ interms of Z. 8

(ii) Show that, under the mapping $w=\frac{i-z}{i+z}$, the image of the circle $x^2+y^2<1$, is the entire half of the w-plane to the right of the imaginary axis. 8

14. (a) Evaluate $\int_C \frac{d\theta}{a+b\cos\theta}$ where a>|b|.

(OR)

14 (b) (i) obtain the Laurent's expansion for $\frac{(z-2)(z+2)}{(z+1)(z+4)}$ which are valid in $1<|z|<4$ and $|z|>4$. 8

(ii) Using Cauchy's integral formula, find the value of $\int_C \frac{z+4}{z^2+2z+5}dz$ where C is the circle $|z+1-i|=2$. 8

15. (a) (i) Find $L\left[\frac{e^{at}-\cos 6t}{t}\right]$ and $L\left[t\cdot e^{-t}\sin t\right]$. 8

(ii) Using Laplace transform solve $y''+4y'+3y=e^{-t};\ y(0)=1,\ y'(0)=0$. 8

(OR)

15. (b) (i) Find the Laplace transform of the rectangular wave given by 8

$$f(t)=\begin{cases}1, & 0<t<b\\ -1, & b<t<2b\end{cases} \text{ with } f(t+2b)=f(t).$$

(ii) State convolution theorem for Laplace transform. 8

Find $L^{-1}\left[\frac{1}{(s^2+4)^2}\right]$ using convolution theorem.

*****THE END*****