Reg. No. : [| | | | | | | | | | |]

Question Paper Code : 27324

B.E./B.Tech. DEGREE EXAMINATION, NOVEMBER/DECEMBER 2015.

Second Semester

Civil Engineering

MA 6251 — MATHEMATICS — II

(Common to all branches except Marine Engineering)

(Regulations 2013)

Time : Three hours | Maximum : 100 marks

Answer ALL questions.

PART A — (10 × 2 = 20 marks)

1. Prove that $3x^2y\vec{i} + (yz - 3xy^2)\vec{j} - \frac{z^2}{2}\vec{k}$ is a solenoidal vector.

2. State Green's theorem.

3. Find the particular integral of $(D^2 - 4D)y = e^x x$.

4. Transform $x^2y''' - 3xy'' = \frac{\sin(\log x)}{x}$ into a differential equation with constant coefficients.

5. State final value theorem on Laplace transform.

6. Find $L^{-1}\left(\frac{s+2}{s^2+4s+8}\right)$.

7. Prove that $w = \sin 2z$ is an analytic function.

8. Define conformal mapping.

9. State Cauchy's integral theorem.

10. Find the residue of $ze^{-\frac{2}{z}}$ at $z = 0$.

PART B — (5 × 16 = 80 marks)

11. (a) (i) Find the directional derivative of $4x^2z + xy^2z$ at $(1,-1,2)$ in the direction of $2\vec{i} - \vec{j} + 3\vec{k}$. (6)

(ii) Using Stoke's theorem evaluate $\iint_S curl\,\vec{f}.\vec{n}\,ds$ given $\vec{f} = y^2\vec{i} + y\vec{j} - xz\vec{k}$ and S is the upper half of the sphere $x^2 + y^2 + z^2 = a^2$. (10)

Or

(b) (i) Find ∇r^n and hence prove that $\nabla^2 r^n = n(n+1)r^{n-2}$. (6)

(ii) Verify Gauss Divergence theorem for $\vec{F} = 4xz\vec{i} - y^2\vec{j} + yz\vec{k}$ taken over the cube bounded by the planes $x = 0$, $x = 1$, $y = 0$, $y = 1$ $z = 0$ and $z = 1$. (10)

12. (a) (i) Solve : $(D^3 - 2D^2 + 4D - 8)y = e^{2x} + \sin x \cos x$. (8)

(ii) Solve : $\frac{dx}{dt} + y = \sin t$; $\frac{dy}{dt} + x = \cos t$ given that $x = 2$, $y = 0$ when $t = 0$. (8)

Or

(b) (i) Solve : $x^2y'' - 4xy' + 6y = x^2 + \log x$. (8)

(ii) Solve : $y'' + 4y = \cot 2x$, using the method of variation of parameters. (8)

13. (a) (i) Find Laplace transform of $t^2e^{-3t}\cos t$ and $\int_0^t \frac{\sin t}{t}\,dt$. (8)

(ii) Using convolution theorem evaluate $\int_0^t \sin u \cos(t-u)\,du$. (8)

Or

(b) (i) Find the Laplace transform of

$$f(t) = \begin{cases} \frac{4E}{T}t - E; & 0 \le t \le \frac{T}{2} \\ 3E - \frac{4E}{T}t, & \frac{T}{2} \le t \le T \end{cases}$$ and $f(t+T) = f(t)$ and E is a constant. (8)

(ii) Solve using Laplace transform, $x'' - 2x' + x = e^t$ when $x(0) = 2$, $x'(0) = -1$. (8)

14. (a) (i) If $f(z)$ is an analytic function, prove that $\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)|f(z)|^p = p^2$ $|f'(z)|^2|f(z)|^{p-2}$. (8)

(ii) Show that the transformation $w=\frac{1}{z}$ transforms all circles and straight lines in the w-plane into circles or straight lines in the z-plane. Which circles in the z-plane become straight lines in the w-plane and which straight lines transform into other straight lines? (8)

Or

(b) (i) Determine the analytic function $f(z)=u+iv$, given $u-v=\frac{\cos x+\sin x-e^{-y}}{2(\cos x-\cosh y)}$ and $f\left(\frac{\pi}{2}\right)=0$. (8)

(ii) Find the bilinear transformation which maps the points $-i,0,i$ into the points $-1,i,1$ respectively. Into what curve the y-axis is transformed under this transformation? (8)

15. (a) (i) Evaluate $\int_C \frac{\tan\frac{z}{2}}{(z-a)^2}dz$, where $-2<a<2$ and C is the boundary of the square whose sides lie along $x=\pm2$ and $y=\pm2$. (8)

(ii) Evaluate $\int_{-\infty}^{\infty}\frac{\cos x\,dx}{(x^2+a^2)(x^2+b^2)}$ using contour integration given $a>b>0$. (8)

Or

(b) (i) Expand Laurent's series $f(z)=\frac{z}{(z-1)(z-2)}$ valid in $1<|z|<2$ and $|z-1|<1$. (8)

(ii) Evaluate $\int_0^{2\pi}\frac{\cos 3\theta\, d\theta}{5-4\cos\theta}$. (8)