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Reg. No. | | | | | | | | | | | | |

Question Paper Code : 60771

B.E./B.Tech. DEGREE EXAMINATION, NOVEMBER/DECEMBER 2016.

Second Semester

Civil Engineering

MA 2161/MA 22/080030004 — MATHEMATICS — II

(Common to all Branches)

(Regulations 2008)

Time : Three hours — Maximum : 100 marks

Answer ALL questions.

PART A — (10 × 2 = 20 marks)

1. Find the particular integral of $(D-2)^2 = e^{2x}\sin 2x$.

2. Solve: $(D^4+1)y = 0$.

3. Find the unit vector normal to the surface x^2+xy+y^2+xyz at the point $(1,-2,1)$.

4. Prove that $\vec{F} = (2x+yz)\hat{i} + (4y+zx)\hat{j} - (6z-xy)\hat{k}$ is solenoidal.

5. Find the constants a, b and c if $f(z) = x+ay+i(bx+cy)$ is analytic.

6. Find the fixed points of the transformation $w = \dfrac{-z+1}{z+1}$.

7. Evaluate $\displaystyle\int_{|z|=2} \frac{dz}{z^2-7z+12}$.

8. Find the residue of the function $\dfrac{4}{z^4(z-3)}$ at a simple pole.

9. Find $L\left(\dfrac{\sin t}{t}\right)$.

10. Using the initial value theorem, find $\underset{s\to\infty}{Lt}\, sL(f(t))$ for the function $f(t) = e^{-t}\cos t$.

PART B — $(5 \times 16 = 80$ marks)

11. (a) (i) Solve: $(D^2+4)y = x\sin x$. (8)

(ii) Solve: $\dfrac{d^2y}{dx^2} - \dfrac{dy}{dx} = e^x \cos x$ using method of variation of parameter. (8)

Or

(b) (i) Solve: $(1+2x)^2 \dfrac{d^2y}{dx^2} - 6(1+2x)\dfrac{dy}{dx} + 16y = 8(1+2x)^2$. (8)

(ii) Solve: $\dfrac{dx}{dt} + 2x - 3y = t; \dfrac{dy}{dt} - 3x + 2y = e^{2t}$. (8)

12. (a) (i) Verify Stoke's theorem for $\vec{F} = (x^2+y^2)\hat{i} - 2xy\hat{j}$ taken around the rectangle bounded by the lines $x = \pm a, y = 0, y = b$. (10)

(ii) Find the angle between the surfaces $x^2+y^2+z^2 = 9$ and $z = x^2+y^2-3$ at the point $(2, -1, 2)$. (6)

Or

(b) (i) Find the values of the constants a,b,c so that $\vec{F} = (axy + bz^3)\hat{i} + (3x^2 - cz)\hat{j} + (3xz^2 - y)\hat{k}$ may be irrotational. For these values of a,b,c, find the scalar potential of $\vec{F}$. (8)

(ii) Using Gauss divergence theorem evaluate $\iint_S \vec{F}.\hat{n}\,ds$ where $\vec{F} = (x^2 - yz)\hat{i} + (y^2 - zx)\hat{j} + (z^2 - xy)\hat{k}$ and S is the surface formed by $x = 0, x = 1, y = 0, y = 2, z = 0$ and $z = 3$. (8)

13. (a) (i) Prove that $u = x^3 - 3xy^2 + 3x^2 - 3y^2 + 1$ is harmonic and also find the analytic function $f(z) = u + iv$. (8)

(ii) Under the transformation $w = \dfrac{1}{z}$, find the image of $|z - 2i| = 2$. (8)

Or

(b) (i) Find the bilinear mapping which maps $-1,0,1$ of the $z-$plane onto $-1,-i,1$ of the $w-$plane. (8)

(ii) Show that an analytic function with constant modulus is constant. (8)

14. (a) (i) Using the method of contour integration evaluate $\int_0^{2\pi} \frac{d\theta}{5+4\sin\theta}$. (8)

(ii) Find the Laurent's series expansion of $\frac{8z+3}{(z+5)(z-2)(z-3)}$ in the region $7<|z+5|<8$. (8)

Or

(b) (i) Using the method of contour integration, evaluate

$$\int_0^{\infty} \frac{x^2}{(x^2+1)(x^2+4)} dx.$$ (8)

(ii) Evaluate $\int_c \frac{z+1}{z^2+2z+4} dz$, where C is the circle $|z+1+i|=2$, using Cauchy's integral formula. (8)

15. (a) (i) Find the Laplace transform of $t^2 e^{2t} \sin t$. (8)

(ii) Using convolution theorem find $L^{-1}\left[\frac{1}{(s^2+2s+5)^2}\right]$. (8)

Or

(b) (i) Find the Laplace transform of the half–wave rectifier given by

$$f(t)=\begin{cases} a\sin wt & ,0<t<\frac{\pi}{w} \\ 0 & ,\frac{\pi}{w}<t<\frac{2\pi}{w} \end{cases} \text{ and } f\left(t+\frac{2\pi}{w}\right)=f(t).$$ (8)

(ii) Solve using Laplace transforms:

$$\frac{d^2y}{dx^2}+4\frac{dy}{dx}+4y=te^{-t}, y(0)=0, y'(0)=1.$$ (8)

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