Reg. No. : | | | | | | | | | | | | |

Question Paper Code : 57030

B.E./B.Tech. DEGREE EXAMINATION, MAY/JUNE 2014.

Second Semester

Civil Engineering

MA 6251 — MATHEMATICS – II

(Common to all Branches Except Marine Engineering)

(Regulation 2013)

Time : Three hours Maximum : 100 marks

Answer ALL questions.

PART A — ($10 \times 2 = 20$ marks)

1. Find the unit normal vector to the surface $x^2 + y^2 = z$ at $(1, -2, 5)$.
2. Prove that $curl(grad\phi) = 0$.
3. Solve the equation $\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + y = 0$.
4. Find the particular integral of the equation $(D^2 - 9)y = e^{-3x}$.
5. Find $L\left[\frac{\sin t}{t}\right]$.
6. Evaluate $L^{-1}\left[\frac{1}{s^2 + 6s + 13}\right]$.
7. Is the function $f(z) = \bar{z}$ analytic?
8. Find the invariant points of $f(z) = z^2$.
9. Evaluate $\int_C \frac{z}{z-2}dz$, where C is (a) $|z| = 1$, (b) $|z| = 3$.
10. State Cauchy's residue theorem

PART B — ($5 \times 16 = 80$ marks)

11. (a) Verify Gauss divergence theorem for $\vec{F} = x^2\vec{i} + y^2\vec{j} + z^2\vec{k}$ taken over the cube bounded by the planes $x = 0, y = 0, z = 0, x = 1, y = 1$ and $z = 1$. (16)

Or

(b) (i) Find the value of n such that the vector $r^n\vec{r}$ is both solenoidal and irrotational. (8)

(ii) Verify Stokes theorem for $\vec{F} = (x^2 - y^2)\vec{i} + 2xy\vec{j}$ in the rectangular region of $z = 0$ plane bounded by the lines $x = 0, y = 0, x = a$ and $y = b$. (8)

12. (a) (i) Solve $(D^2 - 4D + 3)y = \cos 2x + 2x^2$. (8)

(ii) Solve $\frac{d^2y}{dx^2} + a^2y = \tan ax$ using method of variation of parameters. (8)

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(b) (i) Solve $\left(x^2D^2 - xD + 1\right)y = \left(\frac{\log x}{x}\right)^2$. (8)

(ii) Solve the simultaneous equations $\frac{dx}{dt} + 2y = -\sin t$ and $\frac{dy}{dt} - 2x = \cos t$. (8)

13. (a) (i) Find the Laplace transform of $f(t)$, where

$$f(t) = \begin{cases} \sin wt, & for \quad 0 < t < \frac{\pi}{w} \\ 0, & for \quad \frac{\pi}{w} < t < \frac{2\pi}{w} \end{cases}$$

and $f(t + 2\pi) = f(t)$. (8)

(ii) Using convolution theorem find the inverse Laplace transform of $\frac{s^2}{\left(s^2 + a^2\right)\left(s^2 + b^2\right)}$. (8)

Or

(b) (i) Find the Laplace transform of $f(t) = te^{-3t} \cos 2t$. (8)

(ii) Using Laplace transform, solve $\frac{d^2y}{dt^2} + 4y = \sin 2t$, given $y(0) = 3$ and $y'(0) = 4$. (8)

14. (a) (i) Prove that the real and imaginary parts of an analytic function are harmonic functions. (8)

(ii) Find the bilinear transformation that maps $1, i$ and -1 of the z-plane onto 0, 1 and ∞ of the w-plane. (8)

Or

(b) (i) Show that $v = e^{-x}(x \cos y + y \sin y)$ is harmonic function. Hence find the analytic function $f(z) = u + iv$. (8)

(ii) Find the image of $|z + 1| = 1$ under the map $w = \frac{1}{z}$. (8)

15. (a) (i) Obtain the Laurent's series expansion of $f(z) = \frac{z^2 - 1}{(z + 2)(z + 3)}$ in $2 < |z| < 3$. (8)

(ii) Evaluate $\int_0^{2\pi} \frac{d\theta}{13 + 5 \sin \theta}$. (8)

Or

(b) (i) Using Cauchy's residue theorem evaluate $\int_C \frac{(z-1)}{(z-1)^2(z-2)} dz$, where C is $|z - i| = 2$. (8)

(ii) Evaluate by using contour integration $\int_0^{\infty} \frac{dx}{\left(1 + x^2\right)^2}$. (8)
