ANNA UNIVERSITY COIMBATORE

B.E. / B.Tech. DEGREE EXAMINATION – DECEMBER 2008

THIRD SEMESTER

(Common to EEE / ECE / EIE / ICE / MECHATRONICS / TEXTILE TECH(FT) / TEXTILE TECH. / MEDICAL ELECTRONICS)

SM 302 - ENGINEERING MATHEMATICS – III

Time: Three Hours **Maximum: 100 Marks**

PART A – (20 x 2 = 40 Marks)

Answer ALL Questions

1) Write down the Dirichlet's condition for a function to be expanded as a Fourier series.
2) Define the value of the Fourier series of $f(x)$ at a point of discontinuity.
3) If $f(x) = \sin hx$ is defined in $-\pi < x < \pi$, write the values of Fourier coefficients a_0 and a_n.
4) If $x = 2\left[\frac{\sin x}{1} - \frac{\sin 2x}{2} + \frac{\sin 3x}{3} - \frac{\sin 4x}{4} + \cdots\right]$ in $0 < x < \pi$, Prove that $\sum \frac{1}{n^2} = \frac{\pi^2}{6}$.
5) Prove that if $F\{f(x)\} = F(s)$, then $F\{f(x-a)\} = e^{isa}F(s)$.
6) Find the Fourier transform of $f(x)$ defined by $f(x) = \begin{cases} 1, & if\ a < x < b \\ 0, & otherwise \end{cases}$
7) Find the Fourier Cosine transform of. $f(x) = \begin{cases} x, & 0 < x < \pi \\ 0, & x \geq \pi \end{cases}$
8) If $F\{f(x)\} = F(s)$, prove that $F\{x^2 f(x)\} = -\frac{d^2}{ds^2}F(s)$.

9) State Initial and Final value theorem on Z - transform.
10) Find the Z – transform of $\frac{1}{(n+1)}$.
11) Prove that $Z^{-1}\left[\frac{z^2}{(z-a)^2}\right] = (n+1)a^n$.
12) Find the difference equation from $y(n) = (A + nB)2^n$.
13) Form the partial differential equation by eliminating the arbitrary constants 'a' and 'b' from $z = ax^3 + by^3$.
14) Find the Singular solution of $z = px + qy + p^2 + q^2 + 1$.
15) Find the general solution of px + qy = z
16) Find the particular integral of $(D^2\text{-}4DD')z = e^{3x+4y}$
17) Classify the p.d.e $(1+x^2)(4+x^2)u_{xx} + (5+2x^2)u_{xy} + u_{yy} = 0$
18) Write any two assumptions made while deriving the partial differential equation of transverse vibrations of a string.
19) Define steady state. Write the one dimensional heat equation in steady state.
20) Write all the solutions of Laplace equation in Cartesian form, using the method of separation of variables

PART B – (5 x 12 = 60 Marks)

Answer Any FIVE Questions

21) a) Find the Fourier series expansion for $f(x) = x^2$ in $(-\pi, \pi)$ and hence show that $$\frac{1}{1^4}+\frac{1}{2^4}+\frac{1}{3^4}+\ldots\infty=\frac{\pi^4}{90}$$ (8)

b) Obtain the half range cosine series for $f(x) = (x-2)^2$ in the interval (0, 2) (4)

22) Find the Fourier transform of $f(x)=\begin{cases}1-|x| & for\ |x|\le 1\\ 0 & otherwise\end{cases}$

Hence find the values of (i) $\int_0^\infty \frac{\sin^2 t}{t^2}dt$ and (ii) $\int_0^\infty \frac{\sin^4 t}{t^4}dt$ (12)

23) a) Using convolution theorem evaluate the inverse Z-transform of $\frac{z^2}{(z-1)(z-3)}$. (6)

b) Find the inverse Z- transform of $\frac{z^3}{(z-1)^2(z-2)}$. (6)

24) a) Find the Z- transform of (n+1) (n+2) (4)

b) Using Z-transforms, solve **$y(n+2) + 3y(n+1) - 4y(n) = 0$, $n \geq 1$**, given that $y(0) = 3$ and $y(1) = -2$. (8)

25) a) Obtain the complete solution of the equation $z = px + qy - 2\sqrt{pq}$ (6)

b) Solve $(D^2 + DD' - 6D'^2)z = \cos(2x+y)$ (6)

26) a) Solve **$xz\,p + yz\,q = xy$**. (6)

b) Solve $(2D^2 - 5DD' + 2D'^2)z = e^{2x+y}$. (6)

27) A tightly stretched flexible string has its ends fixed at x = 0 and x = *l*. At time t = 0, the string is given a shape defined by $f(x) = kx(l-x)$, where k is a constant, and then released from rest. Find the displacement of any point x of the string at any time t. (12)

28) An infinitely long rectangular plate with insulated surfaces is 10cm wide. The two long edges and one short edge are kept at zero temperature, while the other short edge x = 0 is kept at temperature given by

$$u=\begin{cases}20y & for\ 0\le y\le 5\\ 20(10-y) & for\ 5\le y\le 10\end{cases}$$

Find the steady state temperature at any point in the plate. (12)

*******THE END*******