ANNA UNIVERSITY COIMBATORE

B.E. / B.TECH. DEGREE EXAMINATIONS : MAY / JUNE 2010

REGULATIONS : 2007

THIRD SEMESTER

070030008 - ENGINEERING MATHEMATICS III

(COMMON TO MECHATRONICS / EEE / ECE / MEDICAL ELECTRONICS / ICE / EIE / FASHION / TEXTILE TECH. / TEXTILE CHEMISTRY)

TIME : 3 Hours Max.Marks : 100

PART – A

(20 x 2 = 40 MARKS)

ANSWER ALL QUESTIONS

1. Write down the Dirichlet's conditions for a function to be expanded as a Fourier Series.
2. What is the sum of Fourier series of f(x) = $\begin{cases} 1, 0 < x < \pi \\ 2, \pi < x < 2\pi \end{cases}$ at the point x= π
3. Find the root mean square value of f(x) = x^2 in the interval (0,2π)
4. Find the coefficient b_5 of cos5x in the Fourier cosine series of the function f(x) = sin5x in the interval (0, 2 π).
5. Find Fourier sine transform of e^{-ax}
6. State Convolution theorem on Fourier transforms
7. Write the Fourier transform pair.
8. Prove that if F{f(x)} = F(s) , then F{f(x-a} = e^{isa} F(s)
9. State initial and final value theorem on Z- transform
10. Find Z[n(n-1)]
11. Find Z[4(3) ^{n} + 2(-1)^n]
12. Find the difference equation from y(n) = (A + nB)2^n.
13. Form the partial differential equation by eliminating the arbitrary constants a and b from z= ax +by
14. Solve $\frac{\partial^2 z}{\partial x^2} = \sin y$
15. Solve $\left(D^3 - 3DD'^2 + 2D'^2\right)z = 0$
16. Find the particular integral of $\left(D^2 - 4DD' + 4D'^2\right)z = e^{2x-y}$
17. Classify the partial differential equation $u_{xx} + xu_{yy} = 0$
18. What is the constant a^2 in the wave equation $u_{tt} = a^2 u_{xx}$?
19. Write the three possible solutions of one dimensional wave equation
20. Define Steady state . Write the one dimensional heat equation in steady state.

PART –B

(5 x 12 = 60 Marks)

ANSWER ANY FIVE QUESTIONS

21. Find the Fourier series of $f(x) = \begin{cases} -\pi, -\pi < x < 0 \\ x, 0 < x < \pi \end{cases}$ Deduce that $\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \dots\dots\dots = \frac{\pi^2}{8}$

22. a) Find the Fourier series for f(x) = x^2 in (- π , π) . Hence find $\frac{1}{1^4} + \frac{1}{2^4} + \frac{1}{3^4} + \frac{1}{4^4} + \dots\dots \infty.$ (6)

b) Obtain the Fourier sine series of given f(x) = $\begin{cases} l - x & if\ 0 \le x < l \\ 0 & if\ l \le x < 2l \end{cases}$ (6)

23 . Find the Fourier transform of the function f(x) defined by

f (x) = $\begin{cases} 1 - x^2 & if |x| < 1 \\ 0 & if\ |x| > 1 \end{cases}$

Hence show that (i) $\int_0^\infty \left(\frac{\sin s - s\cos s}{s^3}\right)\cos\left(\frac{s}{2}\right)ds = \frac{3\pi}{16}$

Also show that (ii) $\int_0^\infty \left(\frac{x\cos x - \sin x}{s^3}\right)^2 dx = \frac{\pi}{15}$

24. a) Evaluate $\int_0^\infty \frac{dx}{(x^2+a^2)(x^2+b^2)}$ using transforms (6)

b) Find the Fourier transform of f(x) = $e^{-a^2x^2}$, $a > 0$ (6)

25. a) Find the Z- transform of $\frac{1}{(n+1)(n+2)}$ (6)

b) Using Z – transform, solve y(n+2) – 5 y(n+1) + 6 y(n) = 4^n given that

y(0) = 4 , y(1) = 0. (6)

26. a) Solve $x(y^2+z^2)p + y(z^2+x^2)q = z(y^2-x^2)$ (6)

b) Solve $(D^3 - 7DD'^2 - 6D'^3)z = \sin(x+2y) + e^{3x+y}$ (6)

27. a) Solve $z = px + qy + \sqrt{1+p^2+q^2}$ (6)

b) Solve $(D^2 + D'^2 + 2DD' + 2D + 2D')z = e^{2x+y}$ (6)

28. A rectangular plate with insulated surface is 10 cm wide and so long compared to its width that it may be considered infinite in length without introducing appreciable error. The temperature at short edge y = 0 is given by

$$u(x,0) = \begin{cases} 20x & , for\ 0 \le x \le 5 \\ 20(10-x), & for\ 5 \le x \le 10 \end{cases}$$

and all the other three edges are kept at 0 °c . Find the steady state temperature at any point in the plate

*********THE END *********