ANNA UNIVERSITY COIMBATORE

B.E. / B.TECH. DEGREE EXAMINATIONS : DECEMBER 2009

REGULATIONS : 2007

THIRD SEMESTER

070030004 – ENGINEERING MATHEMATICS III

(COMMON TO CIVIL / AERONAUTICAL / CSE / IT)

TIME : 3 Hours                                      Max.Marks : 100

PART – A

(20 x 2 = 40 MARKS)

ANSWER ALL QUESTIONS

1.  Form the PDE from $Z = f(2x - 6y)$

2.  Find the complete integral of $q = 2px$

3.  Solve $\left(D^3 - 3DD'^2 + 2D'^3\right)Z = 0$

4.  Find the complete solution of $Z = px+qy+p^2 + q^2$

5.  State Dirichlet's conditions for Fourier series

6.  Find the sum of the Fourier series for $f(x) = \begin{cases} x, & 0 < x < 1 \\ 2, & 1 < x < 2 \end{cases}$ at x=1

7.  If the Fourier series for $f(x) = \begin{cases} 0, & 0 < x < \pi \\ \sin x, & \pi < x < 2\pi \end{cases}$ is

$f(x) = -\dfrac{1}{\pi}+\dfrac{2}{\pi}\left[\dfrac{\cos 2x}{1.3}+\dfrac{\cos 4x}{3.5}+\dfrac{\cos 6x}{5.7}+........\right]+\dfrac{1}{2}\sin x$

deduce that $\dfrac{1}{1.3}-\dfrac{1}{3.5}+\dfrac{1}{5.7}-.....\infty = \dfrac{\pi - 2}{4}$

8.  Find the half range sine series for $f(x) = 2$ in $0<x<\pi$

9.  Find the nature of the PDE $4u_{xx} +4u_{xy} + u_{yy} + 2u_x - u_y =0$

10. A string is stretched and fastened to two points $l$ apart. Motion is started by displacing the string into the form $y = y_0 \sin\dfrac{\pi x}{l}$ from which it is released at time $t = 0$. Formulate this problem as the boundary value problem.

11. Write any two solutions of the Laplace equation $u_{xx} + u_{yy} =0$

12. State one dimensional heat equation with the initial and boundary conditions.

13. Define Fourier transform Pair

14. State the convolution Theorem for Fourier Transform.

15. Find Fourier Cosine Transform of $e^{-x}$

16. State Parseval's identity in Fourier Transform.

17. Prove that $z[a^n] = \dfrac{Z}{Z-a}$ is $|z| > |a|$

18. Find Z- Transform of $a^n\, n^3$

19. Find $Z\left[\dfrac{a^n}{n!}\right]$ in Z- Transform

20. Find the Z- Transform of $(n+1)(n+2)$

PART – B

(5 x 12 = 60 MARKS)

ANSWER ANY FIVE QUESTIONS

21. (a)  Solve $x^2\left(y-z\right)p + y^2\left(z-x\right)q = z^2\left(x-y\right)$     (7)

    (b)  Solve $\dfrac{\partial^2 z}{\partial x^2} - 3\dfrac{\partial^2 z}{\partial x\partial y} + 2\dfrac{\partial^2 z}{\partial y^2} = 8\sin(x+3y)$     (5)

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22. Express $f(x) = \frac{1}{2}(\pi - x)$ as a Fourier series with period $2\pi$ to be valid in the interval 0 to $2\pi$. Hence deduce the value of series

$$1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \ldots\ldots$$

23. (a) Find the Fourier sine series for y = x in 0<x<l. Hence show that (6)

$$\sum_{n=1}^{\infty} \frac{1}{n^2} = \frac{\pi^2}{6} \quad \text{by using Parseval's identity.}$$

(b) Find the F.S of f(x) from the table given below upto the third harmonic (6)

| x | 0 | π/3 | 2π/3 | π | 4π/3 | 5π/3 | 2π |
|------|-----|------|------|-----|------|------|-----|
| f(x) | 1.0 | 1.21 | 1.27 | 1.3 | 1.27 | 1.21 | 1.0 |

24. A string is stretched and fastened to two points 'l' apart. Motion is started by displaying the string into the form $k(lx-x^2)$ from which it is released at time t=0. Find the displacement of any point on the string at a distance x from one end at time t.

25. A square plate is bounded by the lines x=0,y=0,x=20 and y=20. It's faces are insulated. The temperature along the upper horizontal edge is given by u(x,20) = x(20-x),when 0<x<20,while the other three edges are kept at 0'c Find the steady state temperature of the plate.

26. Show that the transform of $e^{x^2/2}$ is $e^{s^2/2}$ by finding the Fourier transform of $e^{-a^2 x^2}$, a>0.

27. Find the Fourier sine & cosine transform $e^{-ax}$ ,a>0. Hence deduce that

$$\int_0^{\infty} \frac{\cos mx}{a^2 + x^2} dx \quad \text{and} \quad \int_0^{\infty} \frac{x \sin mx}{a^2 + x^2} dx$$

28. (a) Prove that $Z\left[\frac{1}{n+1}\right] = Z \log \frac{Z}{Z-1}$

(b) Solve $y_{n+1} - 2 y_n = 1$, y = 0

*****THE END*****

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