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B.E./B.Tech. DEGREE EXAMINATION, APRIL/MAY 2003.

Third Semester

MA 231 — MATHEMATICS – III

(Common to All Branches)

Time : Three hours — Maximum : 100 marks

Answer ALL the questions.

PART A — (10 × 2 = 20 marks)

10

1. Find the complete integral of $p+q=pq$ where $p=\frac{\partial z}{\partial x}$ and $q=\frac{\partial z}{\partial y}$.

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2. Solve $\left(D^3-3DD'^2+2D'^3\right)z=0$.

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3. If $f(x)=x^2+x$ is expressed as a Fourier series in the interval $(-2, 2)$, to which value this series converges at $x=2$?

14

4. If the Fourier series corresponding to $f(x)=x$ in the interval $(0, 2\pi)$ is $\frac{a_0}{2}+\sum_1^\infty\{a_n\cos nx+b_n\sin nx\}$, without finding the values of $a_0, a_n; b_n$ find the value of $\frac{a_0^2}{2}+\sum_1^\infty\left(a_n^2+b_n^2\right)$.

5. Classify the following second order partial differential equations :

   (a) $4\frac{\partial^2 u}{\partial x^2}+4\frac{\partial^2 u}{\partial x\partial y}+\frac{\partial^2 u}{\partial y^2}-6\frac{\partial u}{\partial x}-8\frac{\partial u}{\partial y}-16u=0$

   (b) $\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}=\left(\frac{\partial u}{\partial x}\right)^2+\left(\frac{\partial u}{\partial y}\right)^2$.

6. Write any two solutions of the Laplace equation obtained by the method of separation of variables.

7. Obtain the Laplace transform of $\sin 2t - 2t\cos 2t$ in the simplified form.

8. Find $L^{-1}\left[\frac{1}{2}\log_e\left(\frac{s+a}{s-a}\right)\right]$.

9. If $F(s)$ is the Fourier transform of $f(x)$, write the formula for the Fourie transform of $f(x)\cos(ax)$ in terms of F.

10. State the convolution theorem for Fourier transforms.

PART B — ($5 \times 16 = 80$ marks)

11. (i) Prove that the Laplace transform of the triangular wave of period 2 defined by

$$f(t) = t, \quad 0 \le t \le \pi$$
$$= 2\pi - t, \quad \pi \le t \le 2\pi$$

is $\frac{1}{s^2}\tan h\left(\frac{\pi s}{2}\right)$.

(ii) Solve using Laplace transforms :

$$\frac{d^2y}{dt^2} + 4\frac{dy}{dt} + 4y = te^{-t},\ y(0) = 0.\ y'(0) = -1.$$

12. (a) (i) Solve : $z = 1 + p^2 + q^2$.

(ii) Solve : $(y-z)p - (2x+y)q = 2x + z$.

Or

(b) (i) Form the partial differential equation by eliminating the arbitrar functions f and g in $z = x^2 f(y) + y^2 g(x)$.

(ii) Solve : $\left(D^2 - DD' - 20D'^2\right)z = e^{5x+y} + \sin(4x - y)$.

13. (a) (i) Obtain the Fourier series for $f(x) = 1 + x + x^2$ in $(-\pi, \pi)$. Deduc that $\frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \cdots\cdots \infty = \frac{\pi^2}{6}$.

(ii) Obtain the constant term and the first harmonic in the Fourie series expansion for $f(x)$ where $f(x)$ is given in the following table

x :	0	1	2	3	4	5	6	7	8	9	10	11
$f(x)$:	18.0	18.7	17.6	15.0	11.6	8.3	6.0	5.3	6.4	9.0	12.4	15.7

Or

(b) (i) Expand the function $f(x) = x \sin x$ as a Fourier series in the interval $-\pi \le x \le \pi$.

(ii) Obtain the half range cosine series for $f(x) = (x-2)^2$ in the interval $0 < x < 2$. Deduce that $\sum_{1}^{\infty} \frac{1}{(2n-1)^2} = \frac{\pi^2}{8}$.

14. (a) A tightly stretched flexible string has its ends fixed at $x = 0$ and $x = l$. At time $t = 0$, the string is given a shape defined by $f(x) = kx^2(l-x)$, where k is a constant, and then released from rest. Find the displacement of any point x of the string at any time $t > 0$. 1

Or

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(b) The ends A and B of a rod l cm long have the temperatures 40°C and 90°C until steady state prevails. The temperature at A is suddenly raised to 90°C and at the same time that at B is lowered to 40°C. Find the temperature distribution in the rod at time t. Also show that the temperature at the mid point of the rod remains unaltered for all time, regardless of the material of the rod.

15. (a) (i) Find the Fourier transform of $e^{-a|x|}$ if $a > 0$. Deduce that $\int_0^{\infty} \frac{1}{(x^2+a^2)^2} dx = \frac{\pi}{4a^3}$ if $a > 0$.

(ii) Find the Fourier sine transform of $xe^{-x^2/2}$.

Or

(b) (i) Find the Fourier cosine transform of

$$f(x) = 1 - x^2, 0 < x < 1$$
$$= 0, \text{otherwise.}$$

Hence prove that $\int_0^{\infty} \frac{\sin x - x\cos x}{x^3} \cos\left(\frac{x}{2}\right) dx = \frac{3\pi}{16}$.

(ii) Derive the Parseval's identity for Fourier transforms.