A 381

B.E./B.Tech. DEGREE EXAMINATION, NOVEMBER/DECEMBER 2005.

Third Semester

Civil Engineering

MA 231 — MATHEMATICS — III

(Common to all branches **Except** Bio–Medical, Civil Engineering & Computer Based Construction, Fashion Technology, Industrial Bio–Technology and Textile Chemistry)

Time : Three hours | Maximum : 100 marks

Answer ALL questions.

PART A — (10 × 2 = 20 marks)

1. Find the partial differential equation of all planes passing through the origin.

2. Find the particular integral of $(D^3 - 3D^2D' - 4DD'^2 + 12D'^3)z = \sin(x + 2y)$.

3. Does $f(x) = \tan x$ possess a Fourier expansion?

4. State Parseval's Theorem on Fourier series.

5. In the diffusion equation $\frac{\partial u}{\partial t} = \alpha^2 \frac{\partial^2 u}{\partial x^2}$ what does α^2 stand for?

6. Write the steady state heat flow equation in two dimension in Cartesian and Polar form.

7. Find the Laplace transform of $\frac{1}{\sqrt{t}}$.

8. Find the Laplace transform of a periodic functions $f(t) = e^{-t}$ with period K.

9. Find the Fourier transform of $e^{-\alpha|x|}$, $\alpha > 0$.

10. State convolution theorem on Fourier transforms.

PART B — (5 × 16 = 80 marks)

11. (i) Find the Fourier transform of $f(x) = \begin{cases} 1-x^2 & \text{for } |x| \le 1 \\ 0 & \text{for } |x| > 1. \end{cases}$ H

evaluate $\int_0^\infty \left(\frac{x \cos x - \sin x}{x^3} \right)^2 dx$.

(ii) Find the Fourier cosine transform of $e^{-a^2x^2}$. Hence evaluate the Fo

sine transform of $x\, e^{-a^2x^2}$.

12. (a) (i) Solve $(x+y)zp + (x-y)zq = x^2 + y^2$.

(ii) Solve $(D^2 + D'^2 + 2DD' + 2D + 2D' + 1)z = e^{2x+y}$.

Or

(b) (i) Solve $z = px + qy + \sqrt{1 + p^2 + q^2}$.

(ii) Solve $\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial x\, \partial y} - 6\frac{\partial^2 z}{\partial y^2} = y \cos x$.

13. (a) (i) Obtain a Fourier expansion for $\sqrt{1 - \cos x}$ in $-\pi < x < \pi$.

(ii) Obtain the cosine series for $f(x) = x$ in $0 < x < \pi$ and deduce

$$\sum_{n=1}^{\infty} \frac{1}{(2n-1)^4} = \frac{\pi^4}{96}.$$

Or

(b) (i) Find the Fourier series for the function :

$$f(x) = \begin{cases} x & \text{in } 0 < x < 1 \\ 1 - x & \text{in } 1 < x < 2. \end{cases}$$

Deduce that $\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \ldots \infty = \frac{\pi^2}{8}$.

(ii) Find the first fundamental harmonic of the Fourier series of

given by the following table :

x	0	1	2	3	4	5
$f(x)$	9	18	24	28	26	20

14. (a) A taut string of length l has its ends $x = 0$ and $x = l$ fixed. The mid point is taken to a small height h and released from rest at time $t = 0$. Find the displacement $y(x,t)$. (16)

Or

(b) Find the steady state temperature distribution in a rectangular plate of sides a and b insulated at the lateral surface and satisfying the boundary conditions $u(0, y) = u(a, y) = 0$ for $0 \le y \le b$ $u(x, b) = 0$ and $u(x, 0) = x(a - x)$ for $0 \le x \le a$. (16)

15. (a) (i) Find the inverse Laplace transforms of

(1) $\log\left(\dfrac{s+1}{s-1}\right)$

(2) $\dfrac{1}{(s^2+4)^2}$. (2 + 2 = 4)

(ii) Solve the following simultaneous equations by using Laplace transforms :

$$\frac{dx}{dt} - y = e^t$$

$$\frac{dy}{dt} + x = \sin t, \text{ given that } x(0) = 1,\ y(0) = 0.$$ (12)

Or

(b) (i) Find the Laplace transform of :

(1) $t^2 e^{-2t} \cos t$ (2) $\dfrac{e^{-at} - e^{-bt}}{t}$. (2 + 2)

(ii) Use Laplace transform method to solve : (12)

$$\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = e^x \text{ with } y = 2,$$

$$\frac{dy}{dx} = -1 \text{ at } x = 0.$$
