Reg. No. : 6 1 4 0 6 1 0 6 0 6 8

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B.E./B.Tech. DEGREE EXAMINATION, NOVEMBER/DECEMBER 2007.

Third Semester

(Regulation 2004)

Civil Engineering

MA 1201 — MATHEMATICS — III

(Common to all branches of B.E./B.Tech. Except Bio-Medical Engineering)
(Common to B.E. (Part-Time) Second Semester Regulation 2005)

Time : Three hours — Maximum : 100 marks

Answer ALL questions.

PART A — ($10 \times 2 = 20$ marks)

1. Form a partial differential equation by eliminating the arbitrary constants a and b from the equation $(x-a)^2 + (y-b)^2 = z^2 \cot^2 \alpha$.

2. Find the complete solution of the partial differential equation $p^2 + q^2 - 4pq = 0$.

3. If

$$f(x) = \begin{cases} \cos x & \text{if } 0 < x < \pi, \\ 50, & \text{if } \pi < x < 2\pi, \end{cases}$$

and $f(x) = f(x + 2\pi)$ for all x, find the sum of the Fourier series of $f(x)$ at $x = \pi$.

4. Find the coefficient b_5 of $\cos 5x$ in the Fourier cosine series of the function $f(x) = \sin 5x$ in the interval $(0, 2\pi)$.

5. Solve the equation $3\frac{\partial u}{\partial x} + 2\frac{\partial u}{\partial y} = 0$, given that $u(x, 0) = 4e^{-x}$ by the method of separation of variables.

6. Write the one dimensional wave equation with initial and boundary conditions in which the initial position of the string is $f(x)$ and the initial velocity imparted at each point x is $g(x)$

7. Write the Fourier transform pair.

8. Find the Fourier sine transform of $f(x) = e^{-ax} (a > 0)$.

9. Express $Z\{f(n+1)\}$ in terms of $\bar{f}(z)$.

10. Find the value of $Z\{f(n)\}$ when $f(n) = na^n$.

PART B — (5 × 16 = 80 marks)

11. (a) (i) Solve the equation

$$(x^2 - y^2 - z^2)p + 2xyq = 2zx.$$

(ii) Solve (8)

$$(D^2 - DD' - 2(D')^2)z = 2x + 3y + e^{3x+4y}.$$

Or

(b) (i) Solve (8)

$$z^2(p^2 + q^2) = x^2 + y^2.$$

(ii) Solve (8)

$$(D^2 + 3DD' - 4(D')^2)z = x + \sin y.$$

12. (a) (i) Find the Fourier series of $f(x) = x^2$ in $0 < x < 2\pi$ and $f(x) = f(x + 2\pi)$ for all x. (8)

(ii) By finding the Fourier cosine series for $f(x) = x$ in $0 < x < \pi$, show that $\frac{\pi^4}{96} = \sum_{n=1}^{\infty} \frac{1}{(2n-1)^4}$. (8)

Or

(b) (i) Obtain the Fourier series of $f(x)$ of period $2l$ given

$$f(x) = \begin{cases} l - x, & 0 \le x < l \\ 0, & l \le x < 2l. \end{cases}$$ (8)

(ii) Find the complex form of the Fourier series of the function $f(x) = e$

when $-\pi < x < \pi$ and $f(x + 2\pi) = f(x)$. (

13. (a) A tightly stretched string with fixed end points $x = 0$ and $x = l$ is initially at rest in its equilibrium position. If it is set vibrating giving each point a velocity $\lambda x\,(l - x)$, find the displacement $y(x, t)$ of the string at any time t and at a distance x from the zero end. (1

Or

(b) A rectangular plate is bounded by the lines $x = 0, y = 0, x = a, y = b$. Its surfaces are insulated. The temperatures along $x = 0$ and $y = 0$ are kept at 0°C and the others at 100°C. Find the steady state temperature at a point of the plate. (

14. (a) (i) Find the Fourier transform of $f(x)$ defined by

$$f(x) = \begin{cases} 1, & |x| < a, \\ 0, & |x| > a, \end{cases}$$

and hence find the value of $\int_0^\infty \frac{\sin x}{x} dx$.

(ii) By finding the Fourier cosine transform of $f(x) = e^{-ax} (a > 0)$ using Parseval's identity for cosine transform evaluate.

$$\int_0^\infty \frac{dx}{(a^2 \quad x^2)^2}.$$

Or

(b) (i) Find the Fourier transform of $f(x)$ defined by (10)

$$f(x) = \begin{cases} 1 - |x|, \text{ if } |x| < 1, \\ 0, \quad \text{if } |x| > 1, \end{cases}$$

and hence prove that

$$\int_0^\infty \left(\frac{\sin x}{x}\right)^4 dx = \frac{\pi}{3}.$$

(ii) If $F_c(f(x)) = F_c(s)$, prove that $F_c(F_c(x)) = f(s)$. (6)

(In the above F_c stands for the Fourier cosine transform)

15. (a) (i) Find the Z-transform of (8)

$$f(n) = \frac{2n+3}{(n+1)(n+2)}.$$

(ii) Using convolution theorem find (8)

$$Z^{-1}\left(\frac{z^2}{(z+a)^2}\right)$$

Or

(b) (i) Solve the difference equation

$$y(n+3) - 3y(n+1) + 2y(n) = 0$$

given that $y(0) = 4, y(1) = 0$ and $y(2) = 8$, by the method of Z-transform. (8)

(ii) Find

$$Z^{-1}\left(\frac{z^2}{(z+2)(z^2+4)}\right),$$

by the method of partial fractions. (8)