J 253

B.E./B.Tech. DEGREE EXAMINATION, APRIL/MAY 2004.

Third Semester

MA 231 — MATHEMATICS — III

(Common to all branches except Bio-Medical Engineering, Civil Engineering and Computer based construction, Fashion technology, Industrial Bio-technology and Textile Chemistry)

Time : Three hours

Maximum : 100 marks

Answer ALL questions.

PART A — ($10 \times 2 = 20$ marks)

1. Eliminate the arbitrary function f from $z = f\left(\frac{xy}{z}\right)$ and form the partial differential equation.

2. Find the complete integral of $p + q = pq$.

3. Find a Fourier sine series for the function $f(x) = 1;\ 0 < x < \pi$.

4. If the Fourier series for the function

$$f(x) = 0;\quad 0 < x < \pi$$
$$= \sin x\ ;\quad \pi < x < 2\pi$$

is $f(x) = -\frac{1}{\pi} + \frac{2}{\pi}\left[\frac{\cos 2x}{1\cdot 3} + \frac{\cos 4x}{3\cdot 5} + \frac{\cos 6x}{5\cdot 7} + \ldots\right] + \frac{1}{2}\sin x$

deduce that $\frac{1}{1\cdot 3} - \frac{1}{3\cdot 5} + \frac{1}{5\cdot 7} - \ldots\infty = \frac{\pi - 2}{4}$.

5. Classify the partial differential equation $u_{xx} + xu_{yy} = 0$.

6. A rod 30 cm long has its ends A and B kept at 20°C and 80°C respectively until steady state conditions prevail. Find the steady state temperature in the rod.

7. Does Laplace transform of $\frac{\cos at}{t}$ exist? Justify.

8. Find the inverse Laplace transform of $\frac{1}{s(s-a)}$.

9. Solve the integral equation $\int_0^\infty f(x) \cos \lambda x \, dx = e^{-\lambda}$.

10. Find the Fourier transform of $f(x)$ if

$$f(x) = \begin{cases} 1; & |x| < a \\ 0; & |x| > a > 0 \end{cases}$$

PART B — (5 × 16 = 80 marks)

11. (i) Solve $x(y-z)p + y(z-x)q = z(x-y)$

(ii) Solve $(D^3 - 7DD'^2 - 6D'^3)z = \sin(x+2y) + e^{2x+y}$.

12. (a) (i) Determine the Fourier expansion of $f(x) = x$ in the interval $-\pi < x < \pi$.

(ii) Find the half range cosine series for $x \sin x$ in $(0, \pi)$.

Or

(b) (i) Obtain the Fourier series for the function

$$\begin{aligned} f(x) &= \pi x; \ 0 \le x \le 1 \\ &= \pi(2-x); \ 1 \le x \le 2 \end{aligned}$$

(ii) Find the Fourier series of period 2π for the function

$$f(x) = \begin{cases} 1 & \text{in } (0, \pi) \\ 2 & \text{in } (\pi, 2\pi) \end{cases}$$

and hence find the sum of the series $\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \dots\infty$.

13. (a) A string is stretched between two fixed points at a distance $2l$ apart and the points of the string are given initial velocities v where

$$v = \frac{cx}{l} \text{ in } 0 < x < l$$

$$= \frac{c}{l}(2l - x) \text{ in } l < x < 2l$$

x being the distance from one end point. Find the displacement of the string at any subsequent time.

Or

(b) Find the solution of the one dimensional diffusion equation satisfying the boundary conditions :

(i) u is bounded as $t \to \infty$

(ii) $\left[\frac{\partial u}{\partial x}\right]_{x=0} = 0$ for all t

(iii) $\left[\frac{\partial u}{\partial x}\right]_{x=a} = 0$ for all t

(iv) $u(x, 0) = x\,(a - x)\,, 0 < x < a.$

14. (a) (i) Find the Laplace transform of $t^2 e^{2t} \cos 2t$.

(ii) Find the Laplace transform of $f(t)$ if

$$f(t) = e^t\,,\ 0 < t < 2\pi \text{ and } f(t) = f(t + 2\pi).$$

Or

(b) (i) Using convolution theorem find the inverse Laplace transform of $\frac{1}{(s^2 + a^2)^2}$.

(ii) Using Laplace transform solve $\frac{dy}{dt} - 3y = e^{2t}$ subject to $y(0) = 1$.

15. (a) (i) Show that the Fourier transform of

$$f(x) = \begin{cases} a^2 - x^2; & |x| < a \\ 0; & |x| > a > 0 \end{cases}$$

is

$$2\sqrt{\frac{2}{\pi}}\left(\frac{\sin \lambda a - \lambda a \cos \lambda a}{\lambda^3}\right)$$

Hence deduce $\int_0^\infty \frac{\sin t - t \cos t}{t^3} dt = \frac{\pi}{4}$.

(ii) Find the Fourier sine and cosine transform of

$$f(x) = \begin{cases} x; & 0 < x < 1 \\ 2 - x; & 1 < x < 2 \\ 0; & x > 2 \end{cases}$$

Or

(b) (i) If $\bar{f}(\lambda)$ is the Fourier transform of $f(x)$, find the Fourier transform of $f(x-a)$ and $f(ax)$.

(ii) Verify Parseval's theorem of Fourier transform for the function

$$f(x) = \begin{cases} 0; & x < 0 \\ e^{-x}; & x > 0 \end{cases}.$$