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B.E./B.Tech. DEGREE EXAMINATION, NOVEMBER/DECEMBER 2003.

Third Semester

MA 231 — MATHEMATICS — III

Common to All branches except Bio-Medical Engineering, Civil Engineering and Computer Based Construction, Fashion Technology, Industrial Bio–Technology and Textile Chemistry

Time : Three hours Maximum : 100 marks

Answer ALL questions.

PART A — ($10 \times 2 = 20$ marks)

1. Obtain partial differential equation by eliminating arbitrary constants a and b from $(x-a)^2 + (y-b)^2 + z^2 = 1$.

2. Find the general solution of $4\dfrac{\partial^2 z}{\partial x^2} - 12\dfrac{\partial^2 z}{\partial x\,\partial y} + 9\dfrac{\partial^2 z}{\partial y^2} = 0$.

3. State Dirichlet's conditions for a given function to expand in Fourier series.

4. If the Fourier series of the function $f(x) = x + x^2$ in the interval $-\pi < x < \pi$ is

$$\frac{\pi^2}{3} + \sum_{n=1}^{\infty} (-1)^n \left[\frac{4}{n^2}\cos nx - \frac{2}{n}\sin nx\right],$$

then find the value of the infinite series $\dfrac{1}{1^2} + \dfrac{1}{2^2} + \dfrac{1}{3^2} + \cdots$.

5. Classify the following partial differential equations

(a) $y^2 u_{xx} - 2xy\,u_{xy} + x^2 u_{yy} + 2u_x - 3u = 0$

(b) $y^2 u_{xx} + u_{yy} + u_x^2 + u_y^2 + 7 = 0$.

6. An insulated rod of length 60 cm has its ends at A and B maintained at 20°C and 80°C respectively. Find the steady state solution of the rod.

7. If $L[f(t)] = F(s)$, then show that $L[f(at)] = \frac{1}{a} F\left(\frac{s}{a}\right)$.

8. State the convolution theorem for Laplace Transforms.

9. If $F(s)$ is the Fourier transform of $f(x)$, then find the Fourier transform of $f(x-a)$.

10. If $F_s(s)$ is the Fourier sine transform of $f(x)$, show that

$$F_s[f(x)\cos ax] = \frac{1}{2}[F_s(s+a) + F_s(s-a)].$$

PART B — (5 × 16 = 80 marks)

11. Obtain Fourier series for $f(x)$ of period $2l$ and defined as follows

$$\begin{aligned} f(x) &= l - x \quad \text{in } 0 < x \le l \\ &= 0 \quad \text{in } l \le x \le 2l \end{aligned}$$

Hence deduce that

$$1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \cdots = \frac{\pi}{4}$$

$$\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \cdots = \frac{\pi^2}{8}.$$ (16)

12. (a) (i) Find the singular integral of the partial differential equation $z = px + qy + p^2 - q^2$. (6)

(ii) Solve : $(D^2 + 4DD' - 5D'^2)z = 3e^{2x-y} + \sin(x-2y)$. (10)

Or

(b) (i) Find the general solution of

$$(3z-4y)p+(4x-2z)q=2y-3x.$$ (6)

(ii) Solve : $\left(D^2-2DD'+D'^2-3D+3D'+2\right)z=\left(e^{3x}+2e^{-2y}\right)^2$. (10)

13. (a) A tightly stretched string with fixed end points $x=0$ and $x=l$ is initially at rest in its equilibrium position. If it is set vibrating by giving each point a velocity $kx\,(l-x)$. Find the displacement of the string at any time. (16)

Or

(b) A rectangular plate with insulated surface is 10 cm wide so long compared to its width that it may be considered infinite length. If the temperature along short edge $y=0$ is given $u(x,0)=8\sin\frac{\pi x}{10}$ when $0<x<10$, while the two long edges $x=0$ and $x=10$ as well as the other short edge are kept at 0°C, find the steady state temperature function $u(x,y)$. (16)

14. (a) (i) Find the Laplace transform of $te^{2t}\sin 3t$. (4)

(ii) Find the inverse Laplace transform of the following function using convolution theorem (6)

$$\frac{1}{s^3\,(s+5)}$$

(iii) Verify the initial value theorem for the function $1+e^{-2t}$. (6)

Or

(b) Solve the system by using Laplace transform

$$3\frac{dx}{dt}+\frac{dy}{dt}+2x=1$$

$$\frac{dx}{dt}+4\frac{dy}{dt}+3y=0$$

given that $x=0$ and $y=0$ when $t=0$. (16)

15. (a) Find the Fourier transform of $f(x)$ given by

$$f(x) = 1 \text{ for } |x| < 2$$
$$= 0 \text{ for } |x| > 2$$

and hence evaluate $\int_0^\infty \frac{\sin x}{x} dx$ and $\int_0^\infty \left(\frac{\sin x}{x}\right)^2 dx$. (16)

Or

(b) Find Fourier sine and cosine transform of e^{-x} and hence find the Fourier sine transform of $\frac{x}{1+x^2}$ and Fourier cosine transform of $\frac{1}{1+x^2}$. (16)
