(b) Solve $\frac{\partial u}{\partial t} = \frac{1}{2}\frac{\partial^2 u}{\partial x^2}$, $0 \le x \le 12$, $0 \le t \le 12$ with initial and boundary conditions

(i) $u(x, 0) = \frac{1}{4}x(15 - x)$; $0 \le x \le 12$

(ii) $u(0, t) = 0$

(iii) $u(12, t) = 9$, $0 \le t \le 12$

using Schmidt relation by taking $h = k = 3$. (16)

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**Question Paper Code : 23772**

B.E./B.Tech. DEGREE EXAMINATION, NOVEMBER/DECEMBER 2018.

Fourth Semester

Civil Engineering

MA 2264 — NUMERICAL METHODS

(Common to Sixth Semester — Electronics and Communication Engineering, Computer Science and Engineering, Industrial Engineering, Information Technology and Fifth Semester — Polymer Technology, Chemical Engineering, Polymer Technology, Aeronautical Engineering, Mechanical Engineering, Electrical and Electronics Engineering and Mechatronics Engineering)

(Regulations 2008)

Time : Three hours Maximum : 100 marks

Answer ALL questions.

PART A — (10 × 2 = 20 marks)

1. Write down the condition for the convergence of fixed point iteration method.

2. Derive the iterative formula for finding the value of $\frac{1}{N}$ where $N$ is a real number using Newton's method.

3. Find the divided differences of $f(x) = x^3 + x + 2$ for the arguments 1, 3, 6 and 11.

4. Write down the Newton's backward interpolation formula.

5. Write down the Newton-Cote's quadrature formula for equidistant ordinates.

6. Write down the first two derivatives at $x = x_n$ using Newton's backward interpolation formula.

7. Under which condition the fourth order Runge-Kutta method reduces to Simpson's rule?

8. How many values are needed to use Milne's Predictor-Corrector formula prior to the required value?

9. Write down the diagonal five point formula in solving elliptic equations.

10. Write down the Bender-Schmidt explicit formula in solving parabolic equation.

PART B — (5 × 16 = 80 marks)

11. (a) (i) Using Newton's method solve $x\log_{10} x = 12.34$ by starting with $x_0 = 10$ upto 4 decimal places. (8)

(ii) Find the inverse of the matrix $A = \begin{bmatrix} 8 & -4 & 0 \\ -4 & 8 & -4 \\ 0 & -4 & 8 \end{bmatrix}$ by using Gauss-Jordan method. (8)

Or

(b) (i) Solve the system of equations by Gauss elimination method. (8)

$2+3y-z=5$; $4x+4y-3z=3$; $2x-3y+2z=2$.

(ii) Find the numerically largest eigenvalue of $A = \begin{bmatrix} 1 & -3 & 2 \\ 4 & 4 & -1 \\ 6 & 3 & 5 \end{bmatrix}$ by Power method. (8)

12. (a) (i) Find $f(8)$ for the following data using Newton's divided difference formula. (8)

| $x$ | 4 | 5 | 7 | 10 | 11 | 13 |
|---|---|---|---|---|---|---|
| $f(x)$ | 48 | 100 | 294 | 900 | 1210 | 2028 |

(ii) Using Lagrange's interpolation formula, find $y(10)$ given that $y(5)=12$, $y(6)=13$, $y(9)=14$, $y(11)=16$. (8)

Or

(b) Find the cubic spline approximation for the function given below. (16)

| $x$ | 0 | 1 | 2 | 3 |
|---|---|---|---|---|
| $y = f(x)$ | 1 | 2 | 33 | 244 |

Assume $M_0 = 0$, $M_3 = 0$. Also find $y(2.5)$.



13. (a) (i) Find $y'(10)$ from the following table. (8)

| $x$ | 3 | 5 | 11 | 27 | 34 |
|---|---|---|---|---|---|
| $y(x)$ | −13 | 23 | 899 | 17315 | 35606 |

(ii) Evaluate $\int_0^{\pi/2} \sqrt{\sin x}\, dx$ using Simpson's 1/3 rule. (8)

Or

(b) (i) Evaluate $\int_{0.5}^{0.7} e^{-x}\sqrt{x}$ taking 5 ordinates by using Trapezoidal rule. (8)

(ii) Evaluate $\int_0^1\int_0^1 \frac{1}{(1+x+y)}\,dx\,dy$ by using Trapezoidal rule, with step sizes $h = k = 0.5$. (8)

14. (a) Using Runge-Kutta method of fourth order solve $\frac{dy}{dx} = \frac{2xy+e^x}{x^2+xe^x}$ for $x = 1.2$ and 1.4 by using $y_0 = 0$ at $x_0 = 1$. (16)

Or

(b) Given $\frac{dy}{dx} = (x^3+xy^2)e^{-x}$, $y(0)=1$, find $y$ at $x = 0.1, 0.2$ and 0.3 by Taylor's series method and compute $y(0.4)$ by Milne's method. (16)

15. (a) Solve $u_{xx} + u_{yy} = 0$ over the square mesh of side 4 units, satisfying the following boundary conditions : (16)

(i) $u(0, y) = 0$, for $0 \le y \le 4$

(ii) $u(4, y) = 12 + y$, for $0 \le y \le 4$

(iii) $u(x, 0) = 3x$, for $0 \le x \le 4$

(iv) $u(x, 4) = x^2$, for $0 \le x \le 4$

Or