(ii) State the explicit finite difference scheme for the wave equation. Hence, solve the equation $25\frac{\partial^2 u}{\partial x^2} = \frac{\partial^2 u}{\partial t^2}$, subject to $\frac{\partial u}{\partial t}(x, 0) = 0$, $u(0, t) = 0$, $u(5, t) = 0$ and

$u(x, 0) = \begin{cases} 2x, \text{ when } 0 \le x \le 2.5 \\ 10 - 2x, \text{ when } 2.5 \le x \le 5 \end{cases}$. Find $u(x, t)$ for one period of vibration (up to t = 2) taking h = 1. (8)

Reg. No. : [ ][ ][ ][ ][ ][ ][ ][ ][ ][ ][ ][ ]

Question Paper Code : 31268

B.E./B.Tech. DEGREE EXAMINATIONS, APRIL/MAY 2019.

Fourth/Sixth Semester

Civil Engineering

MA 2264 — NUMERICAL METHODS

(Common to all branches)

(Regulation 2008)

Time : Three hours Maximum : 100 marks

Answer ALL questions.

PART A — (10 × 2 = 20 marks)

1. State the fixed point theorem for the iterative method.

2. State the Gauss-Jacobi formula for solving a system of three algebraic equations.

3. Find the interpolating polynomial $y = f(x)$ given the data (4, 1), (6, 3), (8, 8) and (10, 16) using Newton's forward formula.

4. Find the polynomial of degree 3 which passes through the points (–1, –21), (1, 15), (2, 12) and (3,3) using Newton's divided differences.

5. State Newton's forward difference interpolation formula to compute the first order derivative of $y = f(x)$ at $x = x_0$.

6. Apply Gauss two-point formula to evaluate $\int_0^1 \frac{dx}{1+x^2}$ and compare it with exact integration.

7. Use Tailor's series method to find $y(0.1)$ given $\frac{dy}{dx} = x^2 + y^2$, $y(0) = 1$.

8. Use Euler's method to find y at $x = 0.1$ and 0.2, given $\frac{dy}{dx} = x + y$, $y(0) = 1$.

9. Write down Laplace equation and its finite difference analogue and the standard five-point formula.

10. State the finite difference method for solving the wave equation.

PART B — (5 × 16 = 80 marks)

11. (a) (i) Find a positive root of $f(x) = x^3 - 8x + 8 = 0$, other than 2, using iterative method. (8)

(ii) Solve the following system of equations by Gauss-Jordan method :

$x + y + z = 1$; $3x + y - 3z = 5$; $x - 2y - 5z = 10$. (8)

Or

(b) (i) Find a positive root of $f(x) = xe^x - \cos x = 0$ by using Newton-Raphson method. (8)

(ii) Solve the following system by Gauss-Seidal method. (8)

$5x + 2y + z = -12$; $-x + 4y + 2z = 20$; $2x - 3y + 10z = 3$.

12. (a) (i) From the following data, express $y = f(x)$ using Newton's backward interpolation. Hence, find $f(5)$. (8)

x :	0	1	2	3	4
$y = f(x)$:	3	6	11	18	27

(ii) Use Lagrange's interpolation formula to find $y(5)$ given $y(1) = 2$, $y(2) = 4$, $y(3) = 8$, $y(4) = 16$ and $y(7) = 128$. (8)

Or

(b) (i) Using divided difference table, find $f(x)$ which takes the values $f(0) = 1$, $f(1) = 4$, $f(3) = 40$ and $f(4) = 85$. (8)

(ii) Find the cubic spline approximation for the points $(0, 1)$, $(1, 2)$, $(2, 33)$ and $(3, 244)$. Assume that $M(0) = M(3) = 0$. Also, find $y(2.5)$. (8)

13. (a) (i) Find y' and y'' at $x = 2$ given $y(0) = 18$, $y(1) = 10$, $y(3) = -18$ and $y(6) = 40$. (8)

(ii) Evaluate $\int_0^6 y\,dx$ using Trapezoidal and Simpson's 1/3 rules, given the data. (8)

x :	0	1	2	3	4	5	6
y :	0.146	0.161	0.176	0.19	0.204	0.217	0.23

Or

(b) (i) Use Trapezoidal rule to evaluate $\int_1^{1.4}\int_2^{2.4} \frac{dx\,dy}{xy}$. Verify your result with actual integration. (8)

(ii) Use Gaussian three point formula to evaluate $I = \int_1^5 \frac{dz}{z}$. (8)

14. (a) (i) Use Runge-Kutta method of order 4 to find $y(0.1)$ and $y(0.2)$ given $\frac{dy}{dx} = \frac{y - x}{y + x}$, $y(0) = 1$, by taking $h = 0.1$. (8)

(ii) Given $y' = y - x^2$, $y(0) = 1$, $y(0.2) = 1.1218$, $y(0.4) = 1.4682$ and $y(0.6) = 1.7379$. Find $y(0.8)$ by Adam-Bashforth method. (8)

Or

(b) Given $y' = xy + y^2$, $y(0) = 1$, find $y(0.1)$, $y(0.2)$ and $y(0.3)$ by Runge-Kutta method of 4th order and find $y(0.4)$ by Milne's predictor corrector method. (16)

15. (a) Solve Laplace equation $\nabla^2 u = 0$ over the square mesh of 4 units satisfying the following boundary conditions : $u(0, y) = 0$, $0 \le y \le 4$, $u(4, y) = 12 + y$, $0 \le y \le 4$, $u(x, 0) = 3x$, $0 \le x \le 4$, $u(x, 4) = x^2$, $0 \le x \le 4$, for the interior points using Leibmann's iteration method. (16)

Or

(b) (i) State the Bender-Schmitt formula for one dimensional heat equations. Hence, solve $4\frac{\partial^2 u}{\partial x^2} = \frac{\partial u}{\partial t}$, subject to $u(0,t) = 0$, $u(8, t) = 0$, $u(x, 0) = 4x - x^2/2$. Take $h = 1$ and find $u(x,t)$ up to $t = 5/8$. (8)