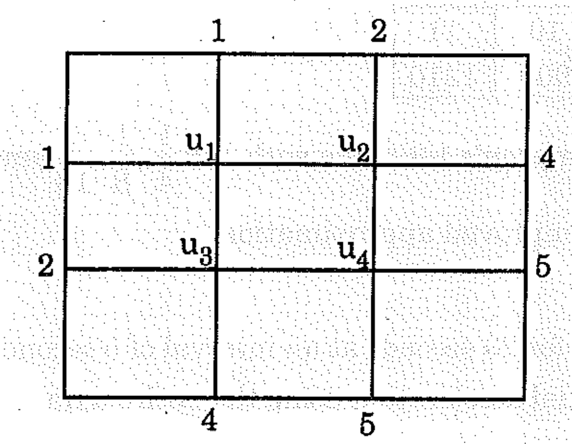
15. a) i) Solve the equation $y'' = x + y$ with boundary conditions $y(0) = y(1) = 0$ with $h = 1/4$. (8)

ii) Evaluate the pivotal values of $u_{tt} = 16u_{xx}$ taking $\Delta x = 1$ upto $t = 1.25$. The boundary conditions are

$u(0, t) = u(5, t) = 0$, $u_t(x, 0) = 0$ and $u(x, 0) = x^2(5 - x)$. (8)

(OR)

b) i) Solve $u_{xx} + u_{yy} = 0$ for the following square mesh. (8)



ii) Using Crank-Nicolson scheme for solving $u_t = u_{xx}$, $0 \le x \le 1$, $t \ge 0$ subject to $u(x, 0) = \sin \pi x$, $0 < x < 1$, $u(0, t) = u(1, t) = 0$, $t > 0$. Take $\Delta x = 1/3$ and $\Delta t = 1/36$. (8)

Reg. No. :

Question Paper Code : 91787

B.E./B.Tech. DEGREE EXAMINATIONS, NOVEMBER/DECEMBER 2019

Fourth/Fifth/Sixth/Seventh Semester

Civil Engineering

MA 6459 – NUMERICAL METHODS

(Common to Aeronautical Engineering/Agriculture Engineering/Electrical and Electronics Engineering/Electronics and Instrumentation Engineering/ Geoinformatics Engineering/Instrumentation and Control Engineering/ Manufacturing Engineering/Mechanical and Automation Engineering/ Petrochemical Engineering/Production Engineering/Chemical Engineering/ Chemical and Electrochemical Engineering/Handloom and Textile Technology/ Petrochemical Technology/Plastic Technology/Polymer Technology/ Textile Chemistry/Textile Technology)

(Regulations 2013)

Time : Three Hours Maximum : 100 Marks

Answer ALL questions.

PART – A (10×2=20 Marks)

1. State the convergence criterion for fixed point iteration.

2. What is diagonal dominance ?

3. Construct the Newton's backward difference table for the following data :

x	1	2	3	4	5
f(x)	2	5	7	14	32

4. State Newton's divided difference interpolation formula.

5. Write down the formula upto the fourth order differences for finding $\frac{d^2y}{dx^2}$ at any point using Newton's forward interpolation formula.

6. State the three-point Gaussian quadrature formula.

7. What is the difference between an initial value problem and a final value problem ?

8. State Adams-Bashforth predictor corrector formulae.

9. State the Bender-Schmidt scheme for solving one-dimensional heat conduction equation.

10. State the standard five-point formula for solving Poisson equation.

PART – B (5×16=80 Marks)

11. a) i) Using Power method, find the numerically largest eigenvalue and the corresponding eigenvector of the matrix $A = \begin{bmatrix} 1 & 6 & 1 \\ 1 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$. Start with an initial approximation for the eigenvector as $\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$. (8)

ii) Using Gauss-Seidel method, solve the system of equations and obtain the solution correct to four decimal places.

$8x - 3y + 2z = 20;\ 4x + 11y - z = 33;\ 6x + 3y + 12z = 35$ (8)

(OR)

b) i) Find the inverse of $A = \begin{pmatrix} 3 & -3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{pmatrix}$ using Gauss Jordan method. (8)

ii) Find the real root of $xe^x - 2 = 0$ correct to 3 decimal places using Newton Raphson method. (8)

12. a) i) Using Lagrange's formula, express the function $\frac{3x^2 + x + 1}{(x-1)(x-2)(x-3)}$ as a sum of partial fractions. (8)

ii) From the following table, estimate the number of students who obtained marks between 40 and 45. (8)

Marks (out of 100)	30-40	40-50	50-60	60-70	70-80
Number of Students	31	42	51	35	31

(OR)

b) Fit a natural cubic spline for the following data and hence evaluate y(1.5) and y′(3) (16)

x	1	2	3	4
y	1	2	5	11

13. a) i) Using Newton's divided difference formula, find the values of f ′(8) and f″(9) from the following data : (8)

x	4	5	7	10	11
f(x)	48	100	294	900	1210

ii) Compute $I = \int_0^{1/2} \frac{x\,dx}{\sin x}$ using Simpson's 1/3[rd] rule with $h = \frac{1}{4}, \frac{1}{8}, \frac{1}{16}$ and apply Romberg's method to obtain improved approximation. (8)

(OR)

b) i) Given the velocity of a particle for 20 seconds at an interval of 5 seconds as in the table below, find the initial acceleration using the entire data : (8)

time t (sec)	0	5	10	15	20
velocity v (m/sec)	0	3	14	69	228

ii) Using trapezoidal rule, evaluate $I = \int_1^2\int_1^2 \frac{dx\,dy}{x+y}$ using four sub-intervals in each direction. (8)

14. a) i) Using the method of Taylor's series upto fourth order, find y at x = 1.1 and x = 1.2 given $\frac{dy}{dx} = x^2 + y^2, y(1) = 2$. (8)

ii) Using Milne's predictor-corrector method, solve $\frac{dy}{dx} = x - y^2, y(0) = 0$ for y(0.8) given y(0.2) = 0.02, y(0.4) = 0.0795 and y(0.6) = 0.1762. (8)

(OR)

b) i) Find the values of y(0.2) and y(0.4) using Runge-Kutta fourth order method with h = 0.2 given $\frac{dy}{dx} = \sqrt{x^2 + y}, y(0) = 0.8$. (10)

ii) Given $\frac{dy}{dx} = y - x^2 + 1, y(0) = 0.5$, find y(0.2) by modified Euler's method. (6)