Reg. No. :

# Question Paper Code : X20785

B.E./B.Tech. DEGREE EXAMINATIONS, NOVEMBER/DECEMBER 2020
Fourth Semester
Civil Engineering
MA 6459 – NUMERICAL METHODS
(Regulations 2013)

Time : Three Hours | Maximum : 100 Marks

Answer ALL questions

PART – A **(10×2=20 Marks)**

1. State the Newton-Raphson formula and the criteria for convergence.

2. Find the dominant Eigen value of $A = \begin{pmatrix} 2 & 3 \\ 5 & 4 \end{pmatrix}$ by power method upto 1 decimal place accuracy. Start with $x^{(0)} = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$.

3. Construct a table of divided difference for the given data :

| | | | | |
|---|---|---|---|---|
| **x** : | 654 | 658 | 659 | 661 |
| **y** : | 2.8156 | 2.8182 | 2.8189 | 2.8202 |

4. Write down the Newton's forward difference interpolation formula for equal intervals.

5. Compare Trapezoidal rule and Simpson's $\frac{1}{3}$ rule for evaluating numerical integration.

6. Change the limits of $\int_0^{\pi/2} \sin x dx$ into (–1, 1).

7. Using Euler's method, find y(0.1) given that $\frac{dy}{dx} = x + y$, y(0) = 1.

8. State Adam's Predictor – Corrector formulae.

9. Classify the following equation :

$$\frac{\partial^2 u}{\partial x^2} + 4\frac{\partial^2 u}{\partial x \partial y} + 4\frac{\partial^2 u}{\partial y^2} - \frac{\partial u}{\partial x} + 2\frac{\partial u}{\partial y} = 0.$$

10. Express $\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}$ in terms of difference approximation.

PART – B **(5×16=80 Marks)**

11. a) i) Find a root of $x \log_{10} x - 1.2 = 0$ using Newton Raphson method correct to three decimal places.

ii) Solve by Gauss Seidal method, the following system :

$20x + y - 2z = 17$, $3x + 20y - z = -18$, $2x - 3y + 20z = 25$.

(OR)

b) i) Find the dominant Eigen values of $A = \begin{bmatrix} 5 & 0 & 1 \\ 0 & -2 & 0 \\ 1 & 0 & 5 \end{bmatrix}$ using power method.

ii) Apply Gauss Jordan method, find the solution of the following system :
$2x - y + 3z = 8$, $-x + 2y + z = 4$, $3x + y - 4z = 0$.

12. a) i) From the given table compute the value of sin 38°.

| **x** : | 0 | 10 | 20 | 30 | 40 |
|---|---|---|---|---|---|
| **sin x** : | 0 | 0.17365 | 0.34202 | 0.5 | 0.64279 |

**(8)**

ii) Using Lagrange's formula find the value of $\log_{10} 323.5$ for the given data :

| **x** : | 321.0 | 322.8 | 324.2 | 325.0 |
|---|---|---|---|---|
| **log x** : | 2.50651 | 2.50893 | 2.51081 | 2.51188 |

**(8)**

b) i) Find the cubic polynomial from the following table using Newton's divided difference formula and hence find f(4).

| **x :** | 0 | 1 | 2 | 5 |
|---|---|---|---|---|
| **y = f(x) :** | 2 | 3 | 12 | 147 |

**(8)**

ii) Find the cubic splines for the following table :

| **x :** | 1 | 2 | 3 |
|---|---|---|---|
| **y :** | –6 | –1 | 16 |

Hence evaluate y(1.5) and y′(2). **(8)**

13. a) The velocity $\upsilon$(km/min) of a moped which starts from rest, is given at fixed intervals of time t(min) as follows :

| t : | 0 | 2 | 4 | 6 | 8 | 10 | 12 |
|---|---|---|---|---|---|---|---|
| $\upsilon$ : | 0 | 10 | 18 | 25 | 29 | 32 | 20 |

i) Estimate approximately the distance covered in 12 minutes, by Simpson's $1/3^{rd}$ rule. **(8)**

ii) Estimate the acceleration at t = 2 seconds. **(8)**

(OR)

b) i) Given that :

| **x :** | 1.0 | 1.1 | 1.2 | 1.3 | 1.4 | 1.5 | 1.6 |
|---|---|---|---|---|---|---|---|
| **y :** | 7.989 | 8.403 | 8.781 | 9.129 | 9.451 | 9.750 | 10.031 |

Find $\frac{dy}{dx}$ at x = 1.1. **(8)**

ii) Use the Romberg method to get an improved estimate of the integral from x = 1.8 to x = 3.4 from the data in table with h = 0.4. **(8)**

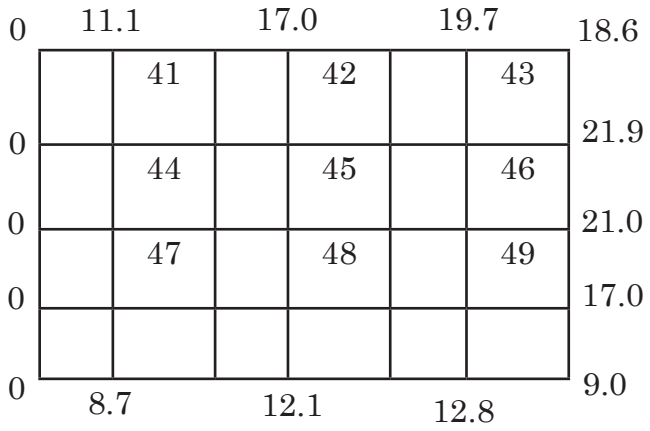
| **x :** | 1.6 | 1.8 | 2.0 | 2.2 | 2.4 | 2.6 |
|---|---|---|---|---|---|---|
| **f(x) :** | 4.953 | 6.050 | 7.389 | 9.025 | 11.023 | 13.464 |
| **x :** | 2.8 | 3 | 3.2 | 3.4 | 3.6 | 3.8 |
| **f(x) :** | 16.445 | 20.056 | 24.533 | 29.964 | 36.598 | 44.701 |

14. a) Determine the value of y(0.4) using Milne's method given $y' = xy + y^2$, y(0) = 1. Use Taylor's series method to get the values of y(0.1), y(0.2) and y(0.3). **(16)**

(OR)

b) Find y(0.1), y(0.2) and y(0.3) from $y' = x + y^2$, y(0) = 1 by using Runge-Kutta method of fourth order and then find y(0.4) by Adam's method. **(16)**

15. a) Solve the Laplace's equation $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$ at the interior points of the square region given as below : **(16)**




(OR)

b) Given that $\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$, u(0, t) = 0, u(4, t) = 0 and u(x, 0) = $\frac{x}{3}(16 - x^3)$. Find $u_{ij}$; i = 1, 2, 3, 4 and j = 1, 2 by using Crank-Nicholson method. **(16)**