Reg. No. : $\square$

## Question Paper Code : 21776

B.E./B.Tech. DEGREE EXAMINATION, NOVEMBER/DECEMBER 2015.

Sixth Semester
Computer Science and Engineering
MA 2264/MA 41/MA 51/MA 1251/080280026/10177 MA 401/10144 CSE 21/ 10144 ECE 15 - NUMERICAL METHODS
(Common to Sixth Semester - Electronics and Communication Engineering Industrial Engineering and Information Technology, Fifth Semester - Polymer Technology, Chemical Engineering and Polymer Technology, Fourth Semester Aeronautical Engineering, Civil Engineering, Electrical and Electronics Engineering and Mechatronics Engineering)
(Regulations 2008/2009)
Time : Three hours
Maximum : 100 marks
Answer ALL questions.

$$
\text { PART A }-(10 \times 2=20 \text { marks })
$$

1. Derive Newton's algorithm for finding the $p^{\text {th }}$ root of a number $N$, where $N>0$.
2. Explain the procedure involved in the Gauss Jordan elimination method.
3. Show that $\bigwedge_{b c d}^{3}\left(\frac{1}{a}\right)=-\frac{1}{a b c d}$.
4. Derive Newton's forward difference formula by using operator method.
5. State two point Gaussian quadrature formula.
6. Evaluate $\int_{\frac{1}{2}}^{1} \frac{1}{x} d x$ by Trapezoidal rule, dividing the range into 4 equal parts.
7. State the disadvantages of Taylor series method.
8. Using Euler's method find $y(0.1)$, given $\frac{d y}{d x}=x+y, y(0)=1$.
9. Write the finite difference approximations for $y^{\prime \prime}(x), y^{\prime \prime}(x)$.
10. Write the Crank-Nicholson scheme to solve $u_{t}=\alpha^{2} u_{x x}$.

$$
\text { PART B }-(5 \times 16=80 \text { marks })
$$

11. (a) (i) Solve $e^{x}-3 x=0$ by the method of fixed point iteration.
(ii) Solve the following system by Gauss-Seidal iterative procedure :

$$
\begin{equation*}
10 x-5 y-2 z=3,4 x-10 y+3 z=-3, x+6 y+10 z=-3 \tag{8}
\end{equation*}
$$

Or
(b) (i) Using Gauss-Jordan method, find the inverse of $\left[\begin{array}{ccc}2 & 2 & 6 \\ 2 & 6 & -6 \\ 4 & -8 & -8\end{array}\right]$.
(ii) Using power method, find all the eigenvalues of $A=\left(\begin{array}{ccc}5 & 0 & 1 \\ 0 & -2 & 0 \\ 1 & 0 & 5\end{array}\right)$.
12. (a) (i) Find the Lagrange polynomial $f(x)$ satisfying the following data:

$$
\begin{array}{ccccc}
x & 1 & 3 & 5 & 7 \\
y=f(x) & 24 & 120 & 336 & 720 \tag{8}
\end{array}
$$

and hence find $f(4)$.
(ii) From the following table :

$$
\begin{array}{cccc}
x & 1 & 2 & 3 \\
y & -8 & -1 & 18 \tag{8}
\end{array}
$$

Compute $y(1: 5)$ and $y^{\prime}(1)$, using cubic spline.

$$
\mathrm{Or}
$$

(b) (i) Using Newton's divided difference formula determine $f(3)$ from the data:

$$
\begin{array}{lccccc}
x & 0 & 1 & 2 & 4 & 5  \tag{8}\\
f(x) & 1 & 14 & 15 & 5 & 6
\end{array}
$$

(ii) From the following data, find $\theta$ at $x=43$ and $x=84$.

$$
\begin{array}{ccccccc}
x: & 40 & 50 & 60 & 70 & 80 & 90 \\
\theta: & 184 & 204 & 226 & 250 & 276 & 304 \tag{8}
\end{array}
$$

Also express $\theta$ interms of $x$.
13. (a) (i) Find $f^{\prime}(3)$ and $f^{\prime \prime}(3)$ for the following data:

| $x$ | 3.0 | 3.2 | 3.4 | 3.6 | 3.8 | 4.0 |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: |
| $f(x)$ | -14 | -10.032 | -5.296 | -.256 | 6.672 | 14 |

(ii) Evaluate $\int_{1}^{2} \int_{3}^{4} \frac{1}{(x+y)^{2}} d x d y$ taking $h=k=0.5$ by both Trapezoidal rule and Simpson's rule.

## Or

(b) (i) Evaluate $\int_{0}^{2} \frac{x^{2}+2 x+1}{1+(x+1)^{4}} d x$ by Gaussian three point formula.
(ii) Using Romberg's method, evaluate $I=\int_{0}^{1} \frac{d x}{1+x}$ correct to three decimal places and hence evaluate the value of $\log _{e}^{2}$.
14. (a) (i) Using Taylor series method solve $\frac{d y}{d x}=1+x y$ with $y(0)=2$. Find $y(0.1), y(0.2)$ and $y(0.3)$.
(ii) Using Milne's method find $y(4.4)$ given $5 x y^{\prime}+y^{2}-2=0$, given $y(4)=1, y(4.1)=1.0049, y(4.2)=1.0097$ and $y(4.3)=1.0143$.

Or
(b) (i) Find $y(0.8)$ given that $y^{\prime}=y-x^{2}, y(0.6)=1.7379$ by using RungeKutta method of order four. Take $h=0.1$.
(ii) Given $\frac{d y}{d x}=x^{2}(1+y), y(1)=1, y(1.1)=1.233, y(1.2)=1.548$, $y(1.3)=1.979$, evaluate $y(1.4)$ by Adams-Bashforth method.
15. (a) (i) Solve $y^{\prime \prime}-y=0$ with $y(0)=0, y(1)=1$ using finite difference method with $h=.2$.
(ii) Solve numerically $4 u_{x x}=u_{t t}$. with the boundary conditions $u(0, t)=0=u(4, t)$ and the initial conditions $u_{t}(x, 0)=0$ and $u(x, 0)=x(4-x)$, taking $h=1$, for 4 time steps.

Or
(b) (i) Solve $\frac{\partial^{2} u}{\partial x^{2}}+\frac{\partial^{2} u}{\partial y^{2}}=0$ at the nodal points of the following square grid using the boundary values indicated.

(ii) Find the values of the function $u(x, t)$ satisfying the differential equation $u_{t}=4 u_{x x}$ and the boundary condition $u(0, t)=0=u(8, t)$ and $u(x, 0)=4 x-\frac{x^{2}}{2}$ at the point $x=i, x=0,1,2,3,4,5,6,7,8, t=\frac{1}{8} j$, $j=0,1,2,3,4,5$.

