Reg. No. $\square$

## Question Paper Code : 91311

# B.E./B.Tech. DEGREE EXAMINATION, MAY/JUNE 2016 Sixth/Fifth Semester <br> Computer Science and Engineering 080230029/080320009 - NUMERICAL METHODS (Common to Chemical Engineering) <br> (Regulations 2008) 

Time : Three Hours
Maximum : 100 Marks

$$
\begin{gathered}
\text { Answer ALL questions. } \\
\text { PART }-\mathbf{A}(10 \times 2=20 \text { Marks })
\end{gathered}
$$

1. State the condition for the convergence of iteration method.
2. What is the order of convergence of Newton-Raphson method?
3. Find $\Delta^{n} e^{\mathrm{ax}}$.
4. Form the divided difference table for the data $(1,22),(2,30),(4,82)$ and $(7,106)$.
5. Write down the trapezoidal rule to evaluate $\int_{1}^{6} f(x) \mathrm{d} x$ with $\mathrm{h}=0.5$.
6. Use two point quadrature formula to solve $\int_{-1}^{1} \frac{d x}{1+x^{2}}$.
7. Comment on the accuracy of Euler's method.
8. What is an initial-value problem ? How is it different from a boundary-value problem ?
9. State the general rule for the classification of second order equations.
10. What is wave equation ?

## PART - B (5 $\times 16=80$ Marks $)$

11. (a) (i) Find the real root of the equation $x=\frac{1}{2}+\sin x$ by iteration method.
(ii) Using Gauss-Jordan method, find the inverse of the matrix $\left(\begin{array}{ccc}2 & 4 & 3 \\ 0 & 1 & 1 \\ 2 & 3 & -1\end{array}\right)$.

## OR

(b) (i) Solve the system of equations by Gausss-Seidel method correct to three decimal places.

$$
\begin{align*}
& x+y+54 z=100  \tag{8}\\
& 27 x+6 y-z=85 \\
& 6 x+15 y+2 z=72
\end{align*}
$$

(ii) Using power method, find the dominant eigen value and eigen vector of the matrix $\left(\begin{array}{ccc}2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2\end{array}\right)$.
12. (a) The table gives the distance in nautical miles of visible horizon for the given heights in feet above the earth's surface.

| Height $(x):$ | 100 | 150 | 200 | 250 | 300 | 350 | 400 |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Distance $(y):$ | 10.63 | 13.03 | 15.04 | 16.81 | 18.42 | 19.90 | 21.27 |

Find the value of $y$ when
(i) $\mathrm{x}=218 \mathrm{ft}$
(ii) $\mathrm{x}=410 \mathrm{ft}$.

## OR

(b) (i) Find the cubic polynomial which takes the following values:

$$
\begin{array}{llllll}
x & : & 0 & 1 & 2 & 3 \\
f(x): & 1 & 2 & 1 & 10
\end{array}
$$

Evaluate $f(4)$.
(ii) In the table below, the values of $y$ are consecutive terms of a series of which 23.6 is the $6^{\text {th }}$ term. Find the first and tenth terms of the series.

| $x:$ | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $y:$ | 4.8 | 8.4 | 14.5 | 23.6 | 36.2 | 52.8 | 73.9 |

13. (a) (i) Find $f(x)$ from the given data and hence find $f^{\prime}(6)$ :

| $x:$ | 0 | 2 | 3 | 4 | 7 | 9 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $f(x):$ | 4 | 26 | 58 | 112 | 466 | 922 |

(ii) Evaluate $\int_{1}^{3} \frac{\sin x}{x} d x$ using Simpson's $\frac{1}{3}$ rule by taking 10 intervals.

## OR

(b) (i) A river is 45 meter wide. The depth d in meters at a distance $x$ meters from one bank is given in the following data :

| $x$ | 0 | 5 | 10 | 15 | 20 | 25 | 30 | 35 | 40 | 45 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| d | 0 | 3 | 6 | 8 | 7 | 7 | 6 | 4 | 3 | 0 |

Find the cross - section of the river by Simpon's $3 / 8$ rule.
(ii) Evaluate $\mathrm{I}=\int_{1}^{3} \int_{1}^{2} \frac{\mathrm{~d} x \mathrm{dy}}{x \mathrm{y}}$ using trapezoidal rule with $\mathrm{h}=\mathrm{k}=0.5$.
14. (a) (i) Find $y(0.1)$ and $y(0.2)$ correct to four decimal places by Runge-Kutta method given $\frac{\mathrm{dy}}{\mathrm{d} x}=\mathrm{y}-x, \mathrm{y}(0)=2$.
(ii) Given $\frac{d y}{d x}=x^{2}(1+y), y(1)=1, y(1.1)=1.233, y(1.2)=1.5485$, $y(1.3)=1.9789$ find $y(1.4)$ by Adam's Predictor-Corrector method.

## OR

(b) (i) Using Modified Euler's method, compute $y$ at $x=0.1$ and 0.2 given that $\frac{d y}{d x}=x+y^{2}, y(0)=1$.
(ii) Using Milne's method find $y(4.4)$ for $5 x y^{\prime}+y^{2}-2=0$ given $y(4)=1$, $y(4.1)=1.0049, y(4.2)=1.0097, y(4.3)=1.143$.
15. (a) (i) Evaluate the pivotal values of the equation $u_{t t}=16 u_{x x}$, taking $h=1$ and $\mathrm{k}=0.25$ upto $\mathrm{t}=1.25$. The boundary conditions are $\mathrm{u}(0, \mathrm{t})=\mathrm{u}(5, \mathrm{t})=0$, $u_{t}(x, 0)=0$ and $u(x, 0)=x^{2}(5-x)$.
(ii) Solve the Laplace's equation $\mathrm{u}_{x x}+\mathrm{u}_{\mathrm{yy}}=0,0<x, \mathrm{y}<1$ with boundary condition $\mathrm{u}(x, 0)=x^{2}, \mathrm{u}(x, 1)=1,0 \leq x \leq 1$ and $\mathrm{u}(0, \mathrm{y})=\mathrm{y}^{2}, \mathrm{u}(1, \mathrm{y})=1,0$ $\leq y \leq 1$ taking $\Delta x=\frac{1}{3}=\Delta y$.

## OR

(b) (i) Using finite difference method solve $\frac{d^{2} y}{d x^{2}}=y$ in $(0,2)$ given $y(0)=0$, $y(2)=3.63$ subdividing the range of $x$ into 4 equal parts.
(ii) Solve the equation $\frac{\partial u}{\partial t}=\frac{\partial^{2} u}{\partial x^{2}}$ subject to the conditions $u(x, 0)=\sin \pi x$, $0 \leq x \leq 1 ; \mathrm{u}(0, \mathrm{t})=\mathrm{u}(1, \mathrm{t})=0$, using Crank-Nicolson method. Carryout computations for two levels, taking $\mathrm{h}=1 / 3, \mathrm{k}=1 / 36$.

