Reg. No. :

Question Paper Code : 31311

B.E./B.Tech. DEGREE EXAMINATION, NOVEMBER/DECEMBER 2015.

Fifth/Sixth Semester

Computer Science and Engineering

080230029/080320009 — NUMERICAL METHODS

(Common to Chemical Engineering)

(Regulation 2008)

Time : Three hours Maximum : 100 marks

Answer ALL questions.

PART A — (10 × 2 = 20 marks)

1. Evaluate $\sqrt{12}$ by Newton-Raphson formula.

2. Solve by iterative method to determine the real root of $x^3 - 2x^2 - 4 = 0$ lying in (2, 3) corrected to three decimal places.

3. Find $\Delta^n e^{ax}$.

4. Form the divided difference table for the data (1, 22), (2,30), (4,82) and (7, 106).

5. Write down the expression for $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ at $x = x_0$ by Newton's backward difference formula.

6. State the formula for 2-point Gaussian quadrature.

7. List out the various methods for solving differential equations.

8. Classify the equation as elliptic, parabolic and hyperbolic

   $xf_{xx} + yf_{yy} = 0, x > 0, y > 0$.

9. Derive the finite difference equation for $\nabla^2 u = -f(x, y)$.

10. State Crank-Nicolson's difference equation in the general form.

PART B — (5 × 16 = 80 marks)

11. (a) (i) Find the root which lies between 2 and 3 correct to 3 decimals of the equation $x^3 - 5x - 7 = 0$ using the method of false position. (8)

(ii) Solve the following system of equations by Gauss-Jordan method : (8)

$$x + 5y + z = 14$$
$$2x + y + 3z = 13$$
$$3x + y + 4z = 17$$

Or

(b) (i) Solve the following system of equations using Gauss-Seidel iteration method (8)

$$x + y + 54z = 110$$
$$27x + 6y - z = 85$$
$$6x + 15y + 2z = 72$$

(ii) Using Power method, find numerically largest eigenvalue and the corresponding eigenvector of the matrix (8)

$$\begin{bmatrix} 1 & -3 & 2 \\ 4 & 4 & -1 \\ 6 & 3 & 5 \end{bmatrix}.$$

12. (a) (i) Using Newton's forward formula obtain a polynomial of degree 4 for the following data

$y(1) = 1, y(2) = -1, y(3) = 1, y(4) = -1, y(5) = 1$. (8)

(ii) Use Lagrange's formula to find the cubic curve that passes through (–1, –8), (1, 3) (2, 1) and (3, 2). (8)

Or

(b) (i) If $f(x) = \frac{1}{x^2}$ find the divided differences $f(x_0, x_1), f(x_0, x_1, x_2)$ and $f(x_0, x_1, x_2, x_3)$. (8)

(ii) Using Newton's divided differences find $f(x)$ given $f(0) = 2, f(1) = 3, f(2) = 12, f(5) = 147$. (8)

13. (a) (i) Find $f(x)$ from the given data and hence find $f'(6)$. (8)

x :	0	2	3	4	7	9
$f(x)$:	4	26	58	112	466	922

(ii) Evaluate $\int_1^3 \frac{\sin x}{x} dx$ using Simpson's $\frac{1}{3}$ rule by taking 10 intervals. (8)

Or

(b) (i) A river is 45 meter wide. The depth d in meters at a distance x meters from one bank is given in the following data :

x :	0	5	10	15	20	25	30	35	40	45
d :	0	3	6	8	7	7	6	4	3	0

Find the cross — section of the river by Simpon's 3/8 rule. (8)

(ii) Evaluate $I = \int_1^3 \int_1^2 \frac{dx\,dy}{xy}$ using trapezoidal rule with $h = k = 0.5$. (8)

14. (a) (i) Using Taylor series method obtain the value of y to three significant figures at $x = 0.1(0.1)0.3$ if y satisfies $y' - 2y = 3e^x, y(0) = 0$.

(ii) Given

$y' = \frac{1}{2}xy, y(0) = 1, y(0.1) = 1.0025, y(0.2) = 1.0101, \quad y(0.3) = 1.0028$, use Adam's method to estimate $y(0.4)$ correct to four decimal places.

Or

(b) (i) Solve $y' = 1 + y^2$ using Runge-Kutta method of order four for $x = 0.2$ given $y(0) = 0$, taking $h = 0.2$.

(ii) Using Milne's predictor and corrector formulae, find $y(4.4)$ given $5xy' + y^2 - 2 = 0, y(4) = 1, y(4.1) = 1.0049, y(4.2) = 1.0097, y(4.3) = 1.0143$.

15. (a) Solve the partial differential equation $\nabla^2 u = -10(x^2 + y^2 + 10)$ over the square with sides $x = 0 = y, x = 3 = y$ with $u = 0$ on the boundary and mesh length = 1.

Or

(b) Solve numerically $4\frac{\partial^2 u}{\partial x^2} = \frac{\partial^2 u}{\partial t^2}$ given that $u(0,t) = 0 = u(4,t); u_t(x,0) = 0$ and $u(x,0) = x(4 - x)$ taking $h = 1$ and $k = \frac{1}{2}$.
