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Question Paper Code : 31152

1.6.13-FN

B.E./B.Tech. DEGREE EXAMINATION, MAY/JUNE 2013.

Sixth Semester

Computer Science and Engineering

080230029 — NUMERICAL METHODS

(Common to 080320009 — Numerical Methods for Fifth Semester Chemical Engineering)

(Regulation 2008)

Time : Three hours Maximum : 100 marks

Answer ALL questions.

PART A — (10 × 2 = 20 marks)

1. Write down the Newton-Raphson iteration formula to solve $f(x)=0$.

2. How the numerically smallest Eigen value is found, for an invertible matrix A?

3. Find $\Delta^n \; e^{ax}$.

4. Form the divided difference table for the data $(1, 22)$, $(2, 30)$, $(4, 82)$ and $(7, 106)$.

5. Evaluate $\int_{-2}^{2} x^4 \, dx$ using trapezoidal rule taking $h = 0.5$.

6. State Romberg iterative formula for composite Simpson's $\frac{1}{3}$ rule.

7. Apply Euler's method to find $y(0.1)$ and $y(0.2)$ given that $\frac{dy}{dx} = y - x$, $y(0) = 2$.

8. Explain the procedure for applying Runge-Kutta method for solving second order ordinary differential equations.

9. Write down Poisson equation and the standard five point finite difference formula to solve them.

10. Give the explicit finite difference scheme for $\frac{\partial^2 u}{\partial x^2} = a\frac{\partial u}{\partial t}$ and obtain the Bender Schmitt recurrence relation.

PART B — (5 × 16 = 80 marks)

11. (a) (i) Find a positive root of $f(x) = x\tan x + 1 = 0$ that lies between 2.5 and 3 using method of false position.

(ii) Find the dominant Eigen value and the correponding Eigen vector for the matrix $A = \begin{bmatrix} 3 & 2 & 4 \\ 2 & 0 & 2 \\ 4 & 2 & 3 \end{bmatrix}$.

Or

(b) (i) Solve the following system of equations by Gauss elimination method :

$3x + 4y + 5z = 18$; $2x - y + 8z = 13$; $5x - 2y + 7z = 20$.

(ii) Solve the following system by Gauss-Seidal method :

$30x - 2y + 3z = 75$; $2x + 2y + 18z = 30$; $x + 17y - 2z = 48$.

12. (a) (i) From the following data, find y at $x = 43$ using Newton's interpolation formula: (8)

x:	40	50	60	70	80	90
y:	184	204	226	250	276	304

(ii) Use Newton's divided difference formula find $f(9)$ given the values $(5, 150)$, $(7, 392)$, $(11, 1452)$, $(13, 2366)$ and $(17, 5202)$. (8)

Or

(b) (i) Use Lagrange's interpolation formula to fit a polynomial $f(x)$ given $f(0) = -12$, $f(1) = 0$, $f(3) = 6$ and $f(4) = 12$. (8)

(ii) Compute the cubic spline in the interval [2, 3] given the data (1, 1), (2, 5), (3, 11) and (4, 8). Assume that $y_0'' = y_3'' = 0$. (8)

13. (a) (i) Find $f(x)$ from the given data and hence find $f'(6)$: (8)

x	0	2	3	4	7	9
$f(x)$	4	26	58	112	466	922

(ii) Evaluate $\int_1^3 \frac{\sin x}{x} dx$ using Simpson's $\frac{1}{3}$ rule by taking 10 intervals. (8)

Or

(b) (i) A river is 45 meter wide. The depth d in meters at a distance x meters from one bank is given in the following data :

x	0	5	10	15	20	25	30	35	40	45
d	0	3	6	8	7	7	6	4	3	0

Find the cross – section of the river by Simpon's 3/8 rule. (8)

(ii) Evaluate $I = \int_1^3 \int_1^2 \frac{dx\, dy}{xy}$ using trapezoidal rule with $h = k = 0.5$. (8)

14. (a) (i) Use Modified Euler's method to find $y(0.1)$ and $y(0.2)$ given that $\frac{dy}{dx} = xy, y(0) = 1$. (8)

(ii) Use Runge-Kutta method of order 4 to find $y(0.1)$ and $y(0.2)$ given $\frac{dy}{dx} = y - x^2, y(0) = 1$, taking $h = 0.1$. (8)

Or

(b) Given $\frac{dy}{dx} = xy + y^2, y(0) = 1$, use Taylor's series method to find the values of $y(0.1), y(0.2), y(0.3)$ and find $y(0.4)$ by Milne's method. (16)

15. (a) (i) Solve $y'' - xy = 0$ for $y(x_i)$ at $x_i = 0, 1/3, 2/3$ given that $y(0) + y'(0) = 1$ and $y(1) = 1$.

(ii) Solve by Leibmann's method $\nabla^2 u = 0$ satisfying the conditions $u(0, y) = 0, u(3, y) = 3y + 9, u(x, 0) = x^2$ and $u(x,3) = 2x^2$, correct to two decimal places.

Or

(b) (i) Use Crank-Nicholson method for the equation $\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$, given $u(x,0) = 10x(5-x), u(0,t) = 0$, $u(5,t) = 0$. Take $\Delta x = 1$ and mesh ratio parameter as 1/2 to find $u(x,t)$ for one time step.

(ii) Solve $16\frac{\partial^2 u}{\partial x^2} = \frac{\partial^2 u}{\partial t^2}, u(0,t) = 0, u(5,t) = 0, \frac{\partial u}{\partial t}(x,0) = 0$ and $u(x,0) = x^2(5-x)$ by finite difference method taking $\Delta x = 1$ and for one period of vibration.
