Reg. No. : | | | | | | | | | | | | |

Question Paper Code : 11347

B.E./B.Tech. DEGREE EXAMINATION, MAY/JUNE 2014.

Sixth/Fifth Semester

Computer Science and Engineering

080230029/080320009 — NUMERICAL METHODS

(Common to Chemical Engineering)

(Regulation 2008)

Time : Three hours — Maximum : 100 marks

Answer ALL questions.

PART A — (10 × 2 = 20 marks)

1. State the condition for the convergence of iteration method.

2. What is the order of convergence of Newton-Raphson method?

3. Write the Lagrange's formula to find $f(x)$ if four sets of values (x_i, y_i), $i = 0, 1, 2, 3$ are given.

4. Construct a table of divided difference for the following data :

x :	0	2	3	5	6
y :	1	19	55	241	415

5. Write down the expression for $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ at $x = x_0$ by Newton's backward difference formula.

6. State the formula for 2-point Gaussian quadrature.

7. Using Euler's method, find $y(0.2)$ given $y' = x + y$ with $y(0) = 1$.

8. Write Milne's Predictor Corrector formula.

9. Give Schmidt explicit formula for one dimensional heat equation.

10. Write down the explicit scheme to solve one-dimensional wave equation.

PART B — (5 × 16 = 80 marks)

11. (a) (i) Find the root which lies between 2 and 3 correct to 3 decimals of the equation $x^3 - 5x - 7 = 0$ using the method of false position. (8)

(ii) Solve the following system of equations by Gauss-Jordan method :

$$x + 5y + z = 14$$
$$2x + y + 3z = 13$$
$$3x + y + 4z = 17.$$
(8)

Or

(b) (i) Solve the following system of equations using Gauss-Seidel iteration method : (8)

$$x + y + 54z = 110$$
$$27x + 6y - z = 85$$
$$6x + 15y + 2z = 72.$$

(ii) Using Power method, find numerically largest eigenvalue and the corresponding eigenvector of the matrix : (8)

$$\begin{bmatrix} 1 & -3 & 2 \\ 4 & 4 & -1 \\ 6 & 3 & 5 \end{bmatrix}.$$

12. (a) (i) Using Lagrange's interpolation formula, fit a polynomial to the following data and hence find the value of y at $x = 2$. (8)

x :	0	1	3	4
y :	–12	0	6	12

(ii) Using Newton's backward interpolation formula, find y when $x = 27$ from the following data : (8)

x :	10	15	20	25	30
y :	35.4	32.2	29.1	26.0	23.1

Or

(b) (i) Fit a natural cubic spline for the following data : (8)

x :	0	1	2	3
y :	1	4	0	–2

(ii) Using Newton's interpolation formula find y at $x = 8$ from the following data : (8)

x :	0	5	10	15	20	25
y :	7	11	14	18	24	32

13. (a) (i) Find first and second derivatives of the functions at the point $x = 1.2$ from the following data : (8)

x :	1	2	3	4	5
y :	0	1	5	6	8

(ii) Evaluate $\int_0^1 \frac{dx}{1+x^2}$ using Trapezoidal rule with $h = 0.2$ and hence obtain the approximate value of π. (8)

Or

(b) (i) By dividing the range into 10 equal parts evaluate $\int_0^\pi \sin x \, dx$ using Simpson's one-third rule. (8)

(ii) Using Romberg's method, evaluate $\int_0^1 \frac{dx}{1+x}$ correct to 4 decimal places and hence find $\log_e 2$. (8)

14. (a) (i) Find $y(0.1)$ and $y(0.2)$ correct to four decimal places by Runge-Kutta method given $\frac{dy}{dx} = y - x$, $y(0) = 2$. (8)

(ii) Given $\frac{dy}{dx} = x^2(1+y)$, $y(1) = 1$, $y(1.1) = 1.233$, $y(1.2) = 1.5485$, $y(1.3) = 1.9789$ find $y(1.4)$ by Adam's Predictor-Corrector method. (8)

Or

(b) (i) Using Modified Euler's method, compute y at $x = 0.1$ and 0.2 given that $\frac{dy}{dx} = x + y^2$, $y(0) = 1$. (8)

(ii) Using Milne's method find $y(4.4)$ for $5xy' + y^2 - 2 = 0$ given $y(4) = 1$, $y(4.1) = 1.0049$, $y(4.2) = 1.0097$, $y(4.3) = 1.143$. (8)

15. (a) (i) Solve $y'' + xy' + y = 3x^2 + 2$, $y(0) = 0$, $y(1) = 1$ with $h = 0.25$ by finite difference method. (8)

(ii) Solve the equation $u_t = u_{xx}$, $0 \le x \le 4$, $t > 0$ using the conditions $u(0, t) = 0$, $u(4, t) = 0$ and $u(x, 0) = \frac{x}{3}(16 - x^2)$ by Crank-Nicolson's method with $h = 1$, $k = 1$. (8)

Or

(b) (i) Solve $u_{tt} = u_{xx}$, $0 < x < 1$, $t > 0$ subject to $u(0, t) = 0$, $u(1, t) = 0$, $\frac{\partial u}{\partial t}(x, 0) = 0$ and $u(x, 0) = 100(x - x^2)$, compute u for 4 time steps taking $h = 0.25$. (8)

(ii) Solve the equation $u_t = u_{xx}$ subject to $u(0, t) = 0$, $u(1, t) = 0$ and $u(x, 0) = 100 \sin \pi x$, $0 < x < 1$ taking $h = 0.2$. Tabulate the values of u for 5 time steps by Bender-Schmidt method with $\lambda = \frac{1}{4}$. (8)
