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Question Paper Code : 80767

B.E./B.Tech. DEGREE EXAMINATIONS, NOVEMBER/DECEMBER 2021.

Fourth Semester

Computer Science and Engineering

MA 2262/MA 44/MA 1252/10177 PQ 401/080250008 — PROBABILITY AND QUEUING THEORY

(Common to Information Technology)

(Regulations 2008/2010)

Time : Three hours   Maximum : 100 marks

Statistical Tables may be permitted

Answer ALL questions.

PART A — (10 × 2 = 20 marks)

1. A continuous random variable X that can assume any value between $x = 2$ and $x = 5$ has a density function given by $f(x) = k(1 + x)$. Find $P(X < 4)$.

2. Identify the random variable and name the distribution it follows, from the following statement :

   "A realtor claims that only 30% of the houses in a certain neighbourhood, are appraised at less than ₹ 20 lakhs. A random sample of 10 houses from that neighbourhood is selected and appraised to check the realtor's claim is acceptable are not".

3. If the joint pdf of the two-dimensional random variable (X, Y) is given by $f(x, y) = K_{xy}e^{-(x^2+y^2)}, x > 0, y > 0$ find the value of K.

4. State central limit theorem.

5. Define wide sense stationary process.

6. If the transition probability matrix (tpm) of a Markov chain is $\begin{pmatrix} 0 & 1 \\ \frac{1}{2} & \frac{1}{2} \end{pmatrix}$, find the steady state distribution of the chain.

7. What are the characteristics of a queuing system?

8. If $\lambda = 4/hr$ and $\mu = 10/hr$ in an (M/M/1): (3/FIFO) queuing system, find the probability that there is no customer in the system.

9. State Jackson's theorem for an open network.

10. What do the letter in the symbolic representation M/G/1 of a queueing model represent?

PART B — (5 × 16 = 80 marks)

11. (a) (i) If the random variable X takes the values 1, 2, 3 and 4 such that $2P(X=1) = 3P(X=2) = P(X=3) = 5P(X=4)$, then find the probability distribution and cumulative distribution function of X. (8)

(ii) Find the MGF of the binomial distribution and hence find its mean. (8)

Or

(b) (i) If the probability that an applicant for a driver's licence will pass the road test on any given trial is 0.8, what is the probability that he will finally pass the test (1) on the 4^{th} trial (2) in fewer than 4 trials? (8)

(ii) The number of monthly breakdowns of a computer is a random variable having a Poisson distribution with mean equal to 1.8. Find the probability that this computer will function for a month (1) without a breakdown (2) with only one breakdown. (8)

12. (a) (i) Let X and Y be random variables having joint density function

$$f(x,y) = \begin{cases} \frac{3}{2}(x^2 + y^2), & 0 \le x \le 1,\ 0 \le y \le 1 \\ 0, & \text{elsewhere} \end{cases}$$

Find the correlation co-efficient γ_{xy}. (8)

(ii) The joint distribution of X and Y is given by $f(x,y) = \dfrac{x+y}{21}$, $x = 1,2,3,\ y = 1,2$. Find the marginal distributions. (8)

Or

(b) (i) If the pdf of 'X' is $f_X(x) = 2x,\ 0 < x < 1$, find the pdf of $Y = 3X + 1$. (8)

(ii) The life time of a certain band of an electric bulb may be considered as a RV with mean 1200 h and SD 250 h. Using central limit theorem, find the probability that the average life time of 60 bulbs exceeds 1250 h. (8)

13. (a) (i) Two random processes $X(t)$ and $Y(t)$ are defined by $X(t) = A\cos\lambda t + B\sin\lambda t$ and $Y(t) = B\cos\lambda t - A\sin\lambda t$. Show that $X(t)$ and $Y(t)$ are jointly wide sense stationary, if A and B are uncorrelated random variables with zero means and the same variances and λ is a constant. (10)

(ii) Prove that difference of two independent Poisson processes is not a Poisson process. (6)

Or

(b) (i) A fair dice is tossed repeatedly. If X_n denotes the maximum of the numbers occurring in the first n tosses, find the transition probability matrix P of the Markov chain $\{X(n)\}$. Also find $P\{X_2 = 6\}$. (8)

(ii) The three – state Markov chain is given by the transition probability matrix $P = \begin{pmatrix} 0 & 2/3 & 1/3 \\ 1/2 & 0 & 1/2 \\ 1/2 & 1/2 & 0 \end{pmatrix}$. Prove that the chain is irreducible and all the states are aperiodic and non null persistent. (8)

14. (a) (i) Derive (1) L_s, average number of customers in the system (2) L_q, average number of customers in the queue for the queuing model $(M/M/1):(N/FIFO)$. (8)

(ii) There are three typists in an office. Each typist can type an average of 6 letters per hour. If letters arrive for being typed at the rate of 15 letters per hour, what fraction of time all the typists will be busy? What is the average number of letters waiting to be typed? (Assume Poisson arrivals and exponential service times) (8)

Or

(b) Customers arrive at a one man barber shop according to a Poisson process with a mean inter arrival time of 20 minutes. Customers spend an average of 15 minutes in the barber chair. The service time is exponentially distributed. If an hour is used as a unit of time, then

(i) What is the probability that a customer need not wait for a hair cut?

(ii) What is the expected number of customer in the barber shop and in the queue?

(iii) How much time can a customer expect to spend in the barber shop?

(iv) Find the average time that a customer spend in the queue.

(v) Estimate the fraction of the day that the customer will be idle?

(vi) What is the probability that there will be 6 or more customers?

(vii) Estimate the percentage of customers who have to wait prior to getting into the barber's chair. (16)

15. (a) Derive Pollaczek-Khintchine formula of an M/G/1 queueing model. (16)

Or

(b) (i) Write a brief note on the open queueing networks. (8)

(ii) A repair facility shared by a large number of machines has 2 series stations with respective service rates of 2 per hour and 3 per hour.

If the average rate of arrival is 1 per hour, find

(1) the average number of machines in the system

(2) the average waiting time in the system

(3) probability that both service stations are idle. (8)
