15. a) In a book shop there are two sections A and B. Customers from outside arrive at the section A at a Poisson rate of 4 per hour and at the section B at a Poisson rate of 3 per hour. The service rates of section A and B are 8 and 10 per hours respectively. A customer upon completion of service at section A is equally likely to go to the section B or to leave the book shop, whereas a customer upon completion of service at B will go the section A with probability 1/3 and will leave the book shop otherwise. Find :

i) The joint steady state probability that there are 4 customers in the section A and 2 customers in the section B

ii) The average number of customers in the book shop and

iii) The average waiting time of customer in the shop.

Assume that there is only one salesman in the each section of shop. **(16)**

(OR)

b) There is only one clerk in a Bank in the loan section. While processing, a clerk gets doubts according to an exponential distribution with a mean of 1/2. To get clarifications, a clerk goes to the Deputy Manager (D.M.) with probability 3/4 and to the Senior Manager (S.M.) with probability 1/4. After completing job with D.M., a clerk goes to S.M. with probability 1/3 and returns to his seat otherwise. Completing the job with S.M., a clerk always returns to his seat. If the D.M. clarifies the doubts and advises a clerk according to an exponential distribution with parameter 1 and the S.M. with parameter 3, compute the probabilities that the clerk is in nodes 1, 2 and 3. Assume that the processing point by clerk is node 1 and service points by D.M. and S.M. are node 2 and node 3 respectively. **(16)**

Reg. No. :

Question Paper Code : 52766

B.E./B.Tech. DEGREE EXAMINATION, NOVEMBER/DECEMBER 2017

Fourth Semester

Computer Science and Engineering

MA 2262 – PROBABILITY AND QUEUEING THEORY

(Common to Information Technology)

(Regulations 2008)

Time : Three Hours Maximum : 100 Marks

Answer ALL questions

PART – A (10×2=20 Marks)

1. If the probability that a target is destroyed on any one shot is 0.5. What is the probability that it would be destroyed on sixth attempt ?
2. Write the mean and variance of uniform distribution.
3. Why are two lines of regression ?
4. If U = X + Y and V = X – Y, how are the joint probability density function of (X, Y) and (U, V) related ?
5. Prove that the Poisson process is not a stationary process.
6. What is meant by steady state distribution of a Markov chain ?
7. Define steady state and transient state queueing systems.
8. Write the Little's formula that hold good for the infinite capacity Poisson queue models.
9. State Pollaczek-Khinchine formula for M/G/1 model.
10. Define a closed Jackson network.

PART – B (5×16=80 Marks)

11. a) i) The diameter of a electric cable, say X, is assumed to be a continuous random variable with probability density function $f(x) = 6x(1 - x), 0 \leq x \leq 1$. Check that f(x) is valid probability density function and determine 'b' such that $P[X < b] = P[X > b]$. (8)

ii) Find the moment generating function of the exponential distribution and hence find its mean and variance. (8)

(OR)

b) i) Find the probability that a person tossing three fair coins get either all heads or all tails for the second time on the fifth trail. (8)

ii) The life time in hours of a certain piece of equipment is continuous random variable with probability density function $f(x) = xe^{-kx}, x > 0$. Find k and $P[X > 2]$. (8)

12. a) i) The joint probability density function of a two dimensional random variable (X, Y) is given by $f(x, y) = \begin{cases} 2 : 0 < y < x < 1 \\ 0 : \text{otherwise} \end{cases}$. Find the marginal density functions of X and Y. Check for independence of X and Y. (8)

ii) The life time of a certain brand of an electric bulb may be considered a random variable with mean 1200 hours and standard deviation 250 hours. Find the probability, using central limit theorem, that the average life time of 60 bulbs exceeds 1400 hours. (8)

(OR)

b) i) If the joint probability density function of a two dimensional continuous random variable (X, Y) is $f(x, y) = e^{-(x+y)}, x \geq 0, y \geq 0$, find $P[X > 1]$ and $P[(X < Y)/(X < 2Y)]$ (8)

ii) Find the most likely price in Bombay corresponding to the price of Rs. 70 at Calcutta from the data given below

	Calcutta	Bombay
Average price	65	67
Standard deviation	2.5	3.5

Coefficient of correlation between the prices of commodities in the two cities is 0.8. (8)

13. a) i) Assume a random process X(t) with four samples $X(t, s_1) = \cos t$, $X(t, s_2) = -\cos t$, $X(t, s_3) = \sin t$, $X(t, s_4) = -\sin t$ which are equally likely. Show that it is a wide sense stationary process. (8)

ii) The region of a salesman consists of three cities A, B and C. He never sells in the same city on successive days. If he sells in city A then the next day he sells in B. However if he sells either in B or C, then the next day he is twice as likely to sell in city A as in the other city. How often does he sell in each of the cities in steady state ? (8)

(OR)

b) i) If customers arrive at a counter in accordance with a Poisson process with a mean rate of 2 per minute, find the probability that the interval between two consecutive arrivals in more than one minute, between one minute and two minutes, four minutes or less. (8)

ii) If $X(t) = \sin(\omega t + \theta)$, where θ is uniformly distributed in $(0, 2\pi)$, show that $\{X(t)\}$ is wide sense stationary process. (8)

14. a) Arrivals at a telephone booth are considered to be Poisson with a mean inter arrival time of 12 minutes. The length of a phone call is assumed to be distributed exponentially with mean 10 minutes.

i) What is the expected number of arrivals in the telephone booth and in the queue ?

ii) Calculate the percentage of time an arrival can walk straight into the telephone booth without having to wait.

iii) How much time can an arrival expect to spend in the telephone booth ?

iv) What is the probability that the waiting time in the system is greater than 30 minutes ? (16)

(OR)

b) A car servicing station has 2 bays where service can be offered simultaneously. Because of space limitations, only 4 cars are accepted for servicing. The arrival pattern is Poisson with 12 cars per day. The service time in both the bays is exponentially distributed with $\mu = 8$ cars per day per bay. Find :

i) The average number of cars in the service station

ii) The average number of cars waiting for service and

iii) The average waiting time a car spends in the system. (16)