Reg. No. :

Question Paper Code : 42769

B.E./B.Tech. DEGREE EXAMINATION, APRIL/MAY 2018

Fourth Semester

Computer Science and Engineering

MA2262 – PROBABILITY AND QUEUEING THEORY

(Common to Information Technology)

(Regulations 2008)

Time : Three Hours

Maximum : 100 Marks

Use of statistical tables may be permitted

Answer ALL questions

PART – A (10×2=20 Marks)

1. If the probability density function of a continuous random variable X is given by

$$f(x)=\begin{cases} ax & ; \ 0 \le x \le 1 \\ a & ; \ 1 \le x \le 2 \\ 3a-ax & ; \ 2 \le x \le 3 \\ 0 & ; \ \text{otherwise} \end{cases}$$

then find the value of 'a'.

2. Suppose that, on an average, in every three pages of a book there is one typographical error. If the number of typographical errors on a single page of the book is a Poisson random variable. What is the probability of at least one error on a specific page of the book ?

3. When will the two regression lines be (a) at right angles (b) coincident ?

4. A small college has 90 male and 30 female professors. An ad-hoc committee of 5 is selected at random to unite the vision and mission of the college. If X and Y are the number of men and women in the committee, respectively, what is the joint probability mass function of X and Y ?

5. Prove that first order stationary random process has a constant mean.

6. Prove that Poisson process is a Markov process.

7. What effect does doubling λ and μ have on L_s and W_s for an (M/M/1) : (FIFO/∞/∞) queuing model ?

8. Write the steady state probabilities for the (M/M/R) : (GD/K/K), $R \le K$ queuing model.

9. State Pollaczek-Khinchine formula.

10. Define closed network of a queuing system.

PART – B (5×16=80 Marks)

11. a) i) Find the MGF of the random variable 'X' having the pdf (8)

$$f(x) = \begin{cases} x, & \text{for } 0 < x < 1 \\ 2 - x, & \text{for } 1 < x < 2 \\ 0, & \text{otherwise} \end{cases}$$

ii) A manufacturer of pins knows that 2% of his products are defective. If he sells pins in boxes of 100 and guarantees that not more than 4 pins will be defective, what is the probability that a box fail to meet the guaranteed quality ? (8)

(OR)

b) i) 6 dice are thrown 729 times. How many times do you expect atleast three dice to show a five (or) a six ? (8)

ii) Suppose that a continuous RV, X follows uniform distribution in the interval (0, 2) and a continuous RV, Y follows exponential distribution with parameter λ. Find λ such that $P(X < 1) = P(Y < 1)$. (8)

12. a) The joint probability density function of a two-dimensional random variable (X, Y) is given by $f(x, y) = xy^2 + \frac{x^2}{8}$; $0 \le x \le 2$; $0 \le y \le 1$. Compute $P(X > 1)$, $P\left(Y < \frac{1}{2}\right)$, $P\left(X > 1 / Y < \frac{1}{2}\right)$, $P\left(Y < \frac{1}{2} / X > 1\right)$; $P(X < Y)$ and $P(X + Y \le 1)$. (16)

(OR)

b) Obtain the equations of the regression lines from the following data. Hence find the coefficient of correlation between X and Y. Also estimate the value of Y when X = 38 and X when Y = 18. (16)

X :	22	26	29	30	31	31	34	35
Y :	20	20	21	29	27	24	27	31

13. a) i) Show that the random process $X(t) = A \cos(\omega t + \theta)$ is a Wide Sense Stationary process if A and ω are constants and θ is a uniformly distributed random variable in $(0, 2\pi)$. (8)

ii) Let $\{X_n\}$ be a Markov chain with state space {0, 1, 2} with initial probability vector $p^{(0)} = (0.7, 0.2, 0.1)$ and the one step transition probability matrix $P = \begin{pmatrix} 0.1 & 0.5 & 0.4 \\ 0.6 & 0.2 & 0.2 \\ 0.3 & 0.4 & 0.3 \end{pmatrix}$. Compute $P(X_2 = 3)$ and $P(X_3 = 2, X_2 = 3, X_1 = 3, X_0 = 2)$. (8)

(OR)

b) i) A random process X(t) is defined by $X(t) = A \cos t + B \sin t$, $-\infty < t < \infty$, where A and B are independent random variables each of which has a value –2 with probability 1/3 and a value 1 with probability 2/3. Show that X(t) is a wide sense stationary process. (8)

ii) If $\{N_1(t)\}$ and $\{N_2(t)\}$ are two independent Poisson processes with parameters λ_1 and λ_2 respectively, then show that $P(N_1(t) = k / N_1(t) + N_2(t) = n) = \binom{n}{k} p^k q^{n-k}$, where $p = \frac{\lambda_1}{\lambda_1 + \lambda_2}$ and $q = \frac{\lambda_2}{\lambda_1 + \lambda_2}$. (8)

14. a) Obtain the steady state probabilities of birth-death process. Also draw the transition graph. (16)

(OR)

b) At a port there are 6 unloading berths and 4 unloading crews. When all the berths are full, arriving ships are diverted to an overflow facility 20 kms down the river. Tankers arrive according to Poisson process with a mean of 1 every 2 hrs. It takes for an unloading crew, on the average, 10 hrs to unload a tanker, the unloading time following an exponential distribution. Find.

i) how may tankers are at the port on the average ?

ii) how long does a tanker spend at the port on the average ?

iii) what is the average arrival rate at the overflow facility ? (16)

15. a) Automatic car wash facility operates with only one bay. Cars arrive according to a Poisson process at the rate of 4 cars per hour and may wait in the facility's parking lot if the bay is busy. The service time for all cars is constant and equal to 10 minutes. Determine L_s, L_q, W_s and W_q. (16)

(OR)

b) Consider a system of two servers where customers from outside the system arrive at sever 1 at a Poisson rate 4 and at server 2 at a Poisson rate 5. The service rates for server 1 and 2 are 8 and 10 respectively. A customer upon completion of service at server 1 is likely to go to server 2 or leave the system; whereas a departure from server 2 will go 25 percent of the time to server 1 and will depart the system otherwise. Determine the limiting probabilities, L_s and W_s. (16)

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