15. (a) A one-man barber shop takes exactly 25 minutes to complete one hair-cut. If customers arrive at the barber shop in a Poisson fashion at an average rate of one every 40 minutes, how long on the average a customer spends in the shop? Also find the average time a customer must wait for service? (16)

Or

(b) A repair facility shared by a large number of machines has 2 sequential stations with respective service rates of 2 per hour and 3 per hour. The cumulative failure rate of all the machines is 1 per hour. Assuming that the system behavior may be approximated by the 2 stage tandem queue, find (16)

(i) the average repair time including the waiting time.

(ii) the probability that both the service stations are idle and

(iii) the bottleneck of the repair facility.

Reg. No. : [ ][ ][ ][ ][ ][ ][ ][ ][ ][ ][ ][ ]

Question Paper Code : 23770

B.E./B.Tech. DEGREE EXAMINATION, NOVEMBER/DECEMBER 2018.

Fourth Semester

Computer Science and Engineering

MA 2262 – PROBABILITY AND QUEUEING THEORY

(Common to Information Technology)

(Regulations 2008)

Time : Three hours Maximum : 100 marks

(Statistical tables to be permitted)

Answer ALL questions.

PART A — (10 × 2 = 20 marks)

1. Given that the pdf of a random variable X is $f(x) = kx, 0 < x < 1$. Find the value of k.

2. If the moment generating function of a random variable X is of the form $(0.4e^t + 0.6)^8$, Evaluate $E(X)$.

3. The joint probability mass function of (X, Y) is given by $p(x, y) = kxy$, $x = 0,1,2$ and $y = 1,2,3$. Find k.

4. The two regression equations of the variables X and Y are $x = 20.1 - 0.5y$ and $y = 11.64 - 0.8x$. Find the means of X and Y.

5. Examine the stationarity of the random process $X(t) = \cos(\lambda t + \theta)$, given that θ is uniformly distributed in $(0, \pi)$ where λ is a constant

6. State the postulates of Poisson process.

7. Define transient and steady state.

8. In M/M/1 model, the arrival rate is 10/ hour and the average waiting time in the system is 3 min. find the average service time in min/customer.

9. Define Series queue with blocking.

10. What is meant by open network?

PART B — (5 × 16 = 80 marks)

11. (a) (i) If X is uniformly distributed over $(-3, 3)$ calculate the probability that (8)

(1) $P(X < 2)$

(2) $P(|X| < 2)$

(3) $P(|X-2| < 2)$

(ii) The time required to repair a machine is exponentially distributed with parameter $\lambda = 1/2$ (8)

(1) What is the probability that the repair time exceeds 2 hours?

(2) What is the probability that the repair takes at least 11 hours, given that its duration exceeds 8 hours?

Or

(b) (i) Fit a Poisson distribution for the following data and hence find the theoretical frequencies. (8)

x	0	1	2	3	4
f	122	60	15	2	1

(ii) In a Normal distribution 31% of the items are under 45 and 8% are over 64. Find the mean and S.D of the distribution. (8)

12. (a) (i) The joint probability mass function of (X, Y) is given by $p(x, y) = \frac{x+y}{21}, x = 1, 2, 3; \ y = 1, 2$. Find the marginal probability distribution and the probability distribution of (X, Y). (8)

(ii) The joint probability density function of a random variable is given by $f(x, y) = \begin{cases} e^{-(x+y)}, & x > 0, y > 0 \\ 0, & otherwise \end{cases}$. Show that X and Y are independent. (8)

Or

(b) Calculate the correlation coefficient for the following data. (16)

X	55	56	58	59	60	60	62
Y	35	38	37	39	44	43	44

13. (a) (i) Consider the random process $X(t) = A \cos \omega t + B \sin \omega t$ where A and B are uncorrelated random variables each with mean 0 and variance 1 and ω is a positive constant. Prove that $\{X(t)\}$ is WSS. (8)

(ii) Show that the matrix $\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1/2 & 1/2 & 0 \end{bmatrix}$ is the transition probability matrix of an irreducible Markov chain. (8)

Or

(b) (i) If {X (t)} is a WSS process with autocorrelation function, $R_{xx}(\tau)$ and if $Y(t) = X(t+a) - X(t-a)$, show that $R_{yy}(\tau) = 2 R_{xx}(\tau + 2a) - R_{xx}(\tau - 2a)$. (8)

(ii) The transition probability matrix of the Markov chain $\{X_n\}, n = 1, 2, 3....$ having 3 states 1,2 and 3 is $P = \begin{bmatrix} 0.1 & 0.5 & 0.4 \\ 0.6 & 0.2 & 0.2 \\ 0.3 & 0.4 & 0.3 \end{bmatrix}$ and the initial distribution is $P^{(0)} = (0.7 \ \ 0.2 \ \ 0.1)$. Find

(1) $P(X_2 = 3)$

(2) $P(X_3 = 2, X_2 = 3, X_1 = 3, X_0 = 2)$ (8)

14. (a) Arrivals at a telephone booth are considered to be Poisson, with an average time 10 minutes between one arrival and the next, the length of a phone call is assumed to be distributed exponentially, with mean 3 minutes (16)

(i) What is the probability that a person arriving at the booth will have to wait?

(ii) Find the average number of units in the system.

(iii) The telephone department will install a second booth when convinced that an arrival would expect waiting for at least 3 minutes for a phone. By how much should the flow of arrivals increase in order to justify a second booth?

Or

(b) (i) In a railway marshalling yard, goods trains arrive at a rate of 30 trains per day. Assuming that the inter-arrival time follows an exponential distribution and the service time distribution is also exponential with an average 36 minutes. Calculate the following: (1) the mean queue size (2) the probability that the queue size exceeds 10 (8)

(ii) A petrol station has 2 pumps. The service time follows the exponential distribution with mean 4 minutes and cars arrive for service in a Poisson process at the rate of 10 cars per hour. (8)

(1) Find the probability that a customer has to wait for service.

(2) What proportion of time the pump remains busy?