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Question Paper Code : X60769

B.E./B.Tech. DEGREE EXAMINATIONS, NOVEMBER/DECEMBER 2020
Fourth Semester
Computer Science and Engineering
MA 2262/MA 44/080250008/10177 PQ 401 – PROBABILITY AND QUEUEING THEORY
(Common to Information Technology)
(Regulations 2008/2010)

Time : Three Hours Maximum : 100 Marks

Answer ALL questions

PART – A **(10×2=20 Marks)**

1. Let X be the random variable which denotes the number of heads in three tosses of a fair coin. Determine the probability mass function of X.

2. A continuous random variable X has a probability density function given by $f(x) = \begin{cases} \frac{3}{4}(2x - x^2), & 0 < x < 2 \\ 0 & \text{otherwise} \end{cases}$. Find $P(X > 1)$.

3. The joint probability mass function of a two dimensional random variable (X, Y) is given by $p(x, y) = k(2x + 3y)$; $x = 0, 1, 2$; $y = 1, 2, 3$. Find the value of k.

4. What do you mean by correlation between two random variables ?

5. If N(t) is the Poisson process, then what can you say about the time we will wait for the first event to occur ? And the time we will wait for the n^{th} event to occur.

6. Is Poisson process stationery ? Justify.

7. What are the characteristics of a queuing system ?

8. What is the probability that a customer has to wait more than 15 minutes to get his service completed in a M/M/1 queuing system, if $\lambda = 6$ per hour and $\mu = 10$ per hour ?

9. Define series queues.

10. Define open network.

PART – B **(5×16=80 Marks)**

11. a) i) A continuous random variable has the pdf $f(x) = kx^4, -1 < x < 0$. Find the value of k and also $P\left\{X > \left(-\frac{1}{2}\right) \middle/ X < \left(-\frac{1}{4}\right)\right\}$. **(8)**

ii) Find the moment generating function of uniform distribution. Hence find its mean and variance. **(8)**

(OR)

b) i) Find the moment generating function and r^{th} moment for the distribution whose pdf is $f(x) = Ke^{-x}, 0 \le x \le \infty$. Hence find the mean and variance. **(8)**

ii) In a large consignment of electric bulbs, 10 percent are defective. A random sample of 20 is taken for inspection. Find the probability that (1) all are good bulbs (2) at most there are 3 defective bulbs (3) exactly there are 3 defective bulbs. **(8)**

12. a) i) Let X and Y be random variables having joint density function

$$f(x, y) = \begin{cases} \frac{3}{2}(x^2 + y^2), & 0 \le x \le 1,\ 0 \le y \le 1 \\ 0, & \text{elsewhere} \end{cases}$$

Find the correlation co-efficient γ_{xy}. **(8)**

ii) The joint distribution of X and Y is given by $f(x, y) = \frac{x+y}{21}$, x = 1, 2, 3, y = 1, 2. Find the marginal distributions. **(8)**

(OR)

b) i) If the pdf of 'X' is $f_X(x) = 2x, 0 < x < 1$, find the pdf of $Y = 3X + 1$. **(8)**

ii) The life time of a certain band of an electric bulb may be considered as a RV with mean 1200 h and SD 250 h. Using central limit theorem, find the probability that the average life time of 60 bulbs exceeds 1250 h. **(8)**

13. a) i) A fair die is tossed repeatedly. The maximum of the first 'n' outcomes is denoted by X_n. Is $\{X_n ; n = 1, 2, \ldots\}$ a Markov chain. If so, find its transition probability matrix, also specify the classes. **(8)**

ii) Show that the process $X(t) = A \cos \lambda t + B \sin \lambda t$ where A and B are random variables, is wide-sense stationary, if $E(A) = E(B) = 0$ and $E(A^2) = E(B^2)$; $E(AB) = 0$. **(8)**

(OR)

b) i) Suppose that customers arrive at a bank according to a Poisson process with a mean rate of 3 per minute; find the probability that during a time interval of 2 mins (1) exactly 4 customers arrive and (2) more than 4 customers arrive. **(8)**

ii) An observer at a lake notices that when fish are caught, only 1 out of 9 trout is caught after another trout, with no other fish between, whereas 10 out of 11 non-trout are caught following non-trout, with no trout between. Assuming that all fish are equally likely to be caught, what fraction of fish in the lake is trout ? **(8)**

14. a) i) A car park has a capacity for 5 cars. The arrival of cars is Poisson at a mean rate of 10 per hour. The length of time each car spends in the car park has negative exponential distribution with mean of 2 minutes. How many cars are in the car park on an average and what is the probability of the newly arriving customer finding the car park full and leaving to park his car elsewhere. **(8)**

ii) In a production shop of a company, the breakdown of the machines is found to be Poisson with the average rate of 3 machines per hour. Breakdown time at one machine costs Rs. 40 per hour to the company. There are two choices before the company for hiring the repairman. One of the repairman is slow but cheap, the other fast but expensive. The slow repairman demands Rs. 20 per hour and will repair the broken down machines exponentially at the rate of 4 per hour. The fast repairman demands Rs. 30 per hour and will repair the machines exponentially at an average rate of 6 per hour. Which repairman should the company hire ? **(8)**

(OR)

b) i) A bank has two tellers working on savings accounts. The first teller handles withdrawals only. The second teller handles deposits only. It has been found that the service time distribution for both deposits and withdrawals are exponential with mean service time of 3 min per customer. Depositors are found to arrive in a Poisson fashion throughout the day with mean arrival rate of 16 per hour. Withdrawals also arrive in a Poisson fashion with mean arrival rate of 14 per hour. What would be the effect on the average waiting time in the queue for the customers if each teller could handle both withdrawals and deposits. What would be the effect, if this could only be accomplished by increasing the service time to 3.5 min ? **(8)**

ii) Consider a bank with two tellers. An average of 80 customers per hour arrive at the bank and wait in the single line for an idle teller. The average time it takes to serve a customer is 1.2 minutes. Assume that inter arrival times and service times are exponential. Determine :

1) The expected number of customers present in the bank
2) The expected length of time a customer spends in the bank
3) The fraction of time that a particular teller is idle. **(8)**

15. a) Derive Pollaczek-Khinchin formula for M/G/1 queue. **(16)**

(OR)

b) i) Consider a two stage tandem queue with external arrival rate λ to node '0'. Let μ_0 and μ_1 be the service rates of the exponential servers at node '0' and '1' respectively. Arrival process is Poisson. Model this system using a Markov chain and obtain the balance equations. **(8)**

ii) Consider two servers. An average of 8 customers per hour arrive from outside at server 1 and an average of 17 customers per hour arrive from outside at server 2. Inter arrival times are exponential. Server 1 can serve at an exponential rate of 20 customers per hour and server 2 can serve at an exponential rate of 30 customers per hour. After completing service at station 1, half the customers leave the system and half go to server 2. After completing service at station 2, 3/4 of the customer complete service and 1/4 return to server 1. Find the expected no. of customers at each server. Find the average time a customer spends in the system. **(8)**
