(ii) A car servicing station has 2 bays where service can be offered simultaneously. Because of space limitation, only 4 cars are accepted for servicing. The arrival pattern is Poisson with 12 cars per day. The service time in both the bays is exponentially distributed with $\mu = 8$ cars per bay per day. Find the average number of cars in the service station, the average number of cars waiting for service. (8)

15. (a) (i) Automatic car wash facility operates with only one bay. Cars arrive according to Poisson distribution with a mean 4 cars per hour and may wait in the car parking lot if the bay is busy. The parking lot is large enough to accommodate any number of cars. If the service time for all cars is constant and equal to 10 minutes. determine the average numbers of cars waiting in the system, the average waiting time of a car in the queue. (8)

(ii) There are two salesmen in a departmental store, one is incharge of billing and receiving the payments and the other is incharge of weighing and delivering the items. After completing the service of billing and payment, the customers reach the shop according to Poisson process at the rate of 5 per hour and both the salesman take 6 minutes each to serve a customer on the average and the service times follow exponential distribution. Find the average number of customers in the shop. (8)

Or

(b) In a network of 3 service stations 1, 2, 3, customers arrive at stations 1 2, 3 from outside, in accordance with Poisson process having rates 5, 10, 15 respectively. The service times at the 3 stations are exponential with respective means 10, 50, 100. A customer completing services at station 1 is equally to go to station 2 or go to station 3 or leaving the station. A customer departing from service at station 2 always goes to station 3. A departure from service at station 3 is equally likely to go station 2 or leave the system. What is average number of customers in the system? What is the average time a customer spends in the system? (16)

Reg. No. :

Question Paper Code : 53251

B.E./B.Tech. DEGREE EXAMINATIONS, APRIL/MAY 2019.

Fourth Semester

Mechanical Engineering (Sandwich)

MA 6453 — PROBABILITY AND QUEUEING THEORY

(Common to Computer Science and Engineering/Information Technology)

(Regulation 2013)

Time : Three hours Maximum : 100 marks

Answer ALL questions.

PART A — (10 × 2 = 20 marks)

1. A coin is tossed three times. If X denotes the number of heads obtained, find the probability distribution of X.

2. State memory less property and which continuous and discrete distributions follow this property?

3. Determine the value of k if $f(x,y)=\begin{cases} kxe^{-y}; & 0<x<2,\ y\geq 0 \\ 0; & \text{otherwise}\end{cases}$ is a joint probability density function of two dimensional random variable (X, Y).

4. When will the two regression lines be (a) at right angles, (b) coincident?

5. Mention various types of random processes.

6. Define n step transition probability in a Markov chain.

7. What are the characteristics of a Queueing system?

8. State the relationship between the average number of customers in the queue and in the system.

9. In (M/G/1) queue model, find the average number of customer in the system if $\lambda=\frac{1}{15}$ per minute, $\mu=\frac{1}{12}$ per minute, $Var(T)=9$ minute.

10. Define series queue model.

PART B — (5 × 16 = 80 marks)

11. (a) (i) If the random variable X takes the values 1,2, 3, 4 such that $2P[X=1]=3P[X=2]=P[X=3]=5P[X=4]$, find the probability distribution and cumulative distribution function of X. (8)

(ii) Find the moment generating function of Binomial distribution and hence find its mean and variance. (8)

Or

(b) (i) The number of accidents in a year to taxi drivers in a city follows a Poisson distribution with mean equal to 3. Out of 1000 taxi drivers, find approximately the number of drivers with no accidents, with more than 3 accidents in a year. (8)

(ii) Trains arrives at a station at 15 minutes intervals starting at 4 am. If a passenger arrives at the station at a random time between 9 am and 9.30 am, find the probability that he has to wait for less than 6 minutes, more than 10 minutes. (8)

12. (a) (i) The joint probability density function of a two dimensional random variable (X, Y) is given by $f(x,y) = \begin{cases} 2 : 0 < x < 1, 0 < y < x \\ 0; \quad \text{elsewhere} \end{cases}$, find the marginal density functions of X and Y. Also find the conditional density function of Y given $X = x$ and the conditional density function of X given $Y = y$. (8)

(ii) Estimate the coefficient of correlation between the two variables x and y from the data given below. (8)

x	1	2	3	4	5	6	7
y	9	8	10	12	11	13	14

Or

(b) (i) Two random variables X and Y have the following joint probability density function $f(x,y)=\begin{cases} 2-x-y; & 0 \le x \le 1, 0 \le y \le 1 \\ 0; & \text{otherwise} \end{cases}$. Find the covariance between X and Y. (8)

(ii) If X and Y are independent random variables each uniformly distributed in (0, 1), find the probability density function of $U = XY$. (8)

13. (a) (i) If $X(t) = \sin(\omega t + Y)$, where Y is uniformly distributed in $(0, 2\pi)$, prove that $\{X(t)\}$ is a wide sense stationary process. (8)

(ii) A radioactive source emits particles at a rate of 5 per minute in accordance with Poisson process. Each particle emitted has a probability 0.6 of being recorded. Find the Probability that 10 particles are recorded in 4 minutes period. (8)

Or

(b) (i) If $\{X(t)\}$ is a wide sense stationary process with autocorrelation function given by $R(\tau) = Ae^{-\alpha|\tau|}$, find the second order moment of the random variable $X(8) - X(5)$. (8)

(ii) The transition probability matrix of a Markov chain $\{X_n\}, n = 1, 2, 3, \ldots$ having three states 1, 2 and 3 is $p = \begin{pmatrix} 0.1 & 0.5 & 0.4 \\ 0.6 & 0.2 & 0.2 \\ 0.3 & 0.4 & 0.3 \end{pmatrix}$ and the initial probability distribution is $p^{(0)} = (0.7 \ \ 0.2 \ \ 0.1)$. Find $P[X_2 = 3]$ and $P[X_3 = 2, X_2 = 3, X_1 = 3, X_0 = 2]$. (8)

14. (a) (i) Customers arrive at a car service station follows a Poisson process at a rate of one per every 10 minutes and service time is an exponential random variable with mean 8 minutes. Find the average number of customers in the system, the average waiting time a customer spends in the system and the average time a customer spends in waiting for service. (8)

(ii) Patients arrive at a clinic according to Poisson distribution at a rate of 30 patients per hour. The waiting room does not accommodate more than 14 patients. Examination time per patient is exponential with mean rate of 20 per hour. Find the effective arrival rate. What is the probability that an arriving patient will not wait? What is the expected waiting time until a patient is discharged from the clinic? (8)

Or

(b) (i) A super market has two girls attending to sales at the counters. If the service time for each customer is exponential with mean 4 minutes and if people arrive in Poisson fashion at the rate of 10 per hour, what is the probability that a customer has to wait for service? What is the expected percentage of idle time for each girl? If the customer has to wait in the queue, what is the expected length of his waiting time? (8)