15. a) Derive Pollaczek – Khintchin formula for the average number of customers in the M/G/1 queueing system. **(16)**

(OR)

b) i) Write a short note on open queueing network. **(8)**

ii) Patients arrive at a clinic in a Poisson fashion at the rate of 3 per hour. Each arriving patients has to pass through two sections. The assistant in the first section take 15 minutes per patient and the doctor in the second section takes nearly 6 minutes per patient. If the service times in the two sections are approximately exponential, find the probability that there are 3 patients in the first sections and 2 patients in the second section. Find also average number of patients in the clinic and the average waiting time of a patient. **(8)**

---

Reg. No. : | | | | | | | | | | | | |

**Question Paper Code : 91786**

B.E./B.Tech. DEGREE EXAMINATIONS, NOVEMBER/DECEMBER 2019

Fourth Semester

Mechanical Engineering (Sandwich)

MA 6453 – PROBABILITY AND QUEUEING THEORY

(Common to Computer Science and Engineering/Information Technology)

(Regulations 2013)

Time : Three Hours Maximum : 100 Marks

Answer ALL questions

PART – A **(10×2=20 Marks)**

1. If the probability density function of a random variable X is $f(x)=\frac{1}{4}$ in $-2<x<2$ find $P(|X|>1)$.

2. If X is a geometric variate, taking values 1, 2, 3, ..., ∞, find P(X is odd).

3. The joint p.d.f. of R.V. (X, Y) is given as $f(x,y)=\begin{cases}\frac{1}{x}, & 0<y<x\le 1\\ 0, & \text{otherwise}\end{cases}$. Find the marginal p.d.f. of Y.

4. Let X and Y be two independent R.Vs with Var(X) = 9 and Var(Y) = 3. Find Var(4X – 2Y + 6).

5. Write the classification of random processes.

6. State any two properties of Poisson process.

7. What do the letters in the symbolic representation (a/b/c) (d/e) of a queueing model represent ?

8. What do you mean by balking and reneging ?

9. Write down Pollaczek-Khintchin formula.

10. What do you mean by bottle neck of a network ?

PART – B (5×16=80 Marks)

11. a) i) A radar system has a probability of 0.1 of detecting a certain target during a single scan. Use Binomial distribution to find the probability that the target will be detected at least 2 times in four consecutive scans. Also compute the probability that the target will be detected at least once in twenty scans. (8)

ii) An electrical firm manufactures light bulbs that have a length of life which is normally distributed with mean $\mu = 800$ hours and standard deviation $\sigma = 40$ hours. Find the probability that a bulb burns between 778 and 834 hours. (8)

(OR)

b) i) The probability of an individual suffering a bad reaction from an injection of a certain antibiotic is 0.001. Out of 2000 individuals, use Poisson distribution to find the probability that exactly three suffer. Also, find the probability of more than two suffer from bad reaction. (8)

ii) Electric trains in a particular route run every half an hour between 12. Midnight and 6 a.m. Using uniform distribution, find the probability that a passenger entering the station at any time between 1.00 a.m. and 1.30 a.m. will have to wait at least twenty minutes. (8)

12. a) Two random variables X and Y have the joint probability density function

$$f(x,y) = \begin{cases} c(4-x-y), & 0 \le x \le 2, 0 \le y \le 2 \\ 0, & \text{elsewhere} \end{cases}.$$

Find the equations of two lines of regression. (16)

(OR)

b) i) The joint distribution of X and Y is given by $f(x,y) = \frac{x+y}{21}$, $x = 1, 2, 3, y = 1, 2$. Find marginal distributions and conditional distributions. (8)

ii) If X and Y are independent random variables with probability density functions $e^{-x}, x \ge 0$ and $e^{-y}, y \ge 0$ respectively, find the density function of $U = \frac{X}{X+Y}$. (8)



13. a) i) Consider a random process $Y(t) = X(t)\cos(w_0 t + \theta)$, where X (t) is wide-sense stationary process, $\theta$ is a uniformly distributed R.V. over $(-\pi, \pi)$ and $w_0$ is a constant. It is assumed that X(t) and $\theta$ are independent. Show that Y(t) is a wide-sense stationary. (8)

ii) Consider a Markov chain $\{X_n ; n = 0, 1, 2, ....\}$ having state space S = {1, 2} and one-step

$$\text{TPM } P = \begin{bmatrix} \frac{4}{10} & \frac{6}{10} \\ \frac{8}{10} & \frac{2}{10} \end{bmatrix}$$

1) Draw a transition diagram.
2) Is the chain irreducible ?
3) Is the state-1 ergodic ? Explain.
4) Is the chain ergodic ? Explain. (8)

(OR)

b) i) Let X(t) and Y(t) be two independent Poisson processes with parameters $\lambda_1$ and $\lambda_2$ respectively. Find

1) $P(X(t) + Y(t)) = n, n = 0, 1, 2, 3, ...,$
2) $P(X(t) - Y(t)) = n, n = 0, \pm 1, \pm 2, ....$ (8)

ii) Find the nature of the states of the Markov process with the transition probability matrix $P = \begin{bmatrix} 0 & 1 & 0 \\ 0.5 & 0 & 0.5 \\ 0 & 1 & 0 \end{bmatrix}$. (8)

14. a) i) Obtain $P_0$ and $P_n$ for the birth and death process. (8)

ii) Arrives at a telephone booth are considered to be Poisson distribution, with an average time of 10 minutes between one arrival and the next. The length of phone call assumed to be distributed exponentially with mean 3 minutes. What is the probability that a person arriving at the booth will have to wait ? And what is the average length of the queue that form from time to time ? (8)

(OR)

b) There are three typists in an office. Each typist can type an average of 6 letters per hour. If letters arrive for being typed at the rate of 15 letters per hour. What is the probability of all the typists will be busy ? What is the average number of letters waiting to be typed ? What is the average time a letter has to be spent for waiting and for being typed ? And what is the probability that a letter will take longer than 20 minutes waiting to be typed and being typed ? (16)