Reg. No. : [ ][ ][ ][ ][ ][ ][ ][ ][ ][ ][ ][ ]

Question Paper Code : 91581

B.E./B.Tech. DEGREE EXAMINATION, NOVEMBER/DECEMBER 2014.

Fourth Semester

Computer Science and Engineering

MA 2262/MA 44/MA 1252/080250008/10177 PQ 401 – PROBABILITY AND QUEUEING THEORY

(Common to Information Technology)

(Regulation 2008/2010)

Time : Three hours | Maximum : 100 marks

Use of statistical tables may be permitted.

Answer ALL questions.

PART A — (10 × 2 = 20 marks)

1. Test whether $f(x)=\begin{cases}|x| ; -1 \le x \le 1 \\ 0 \ ;\text{otherwise}\end{cases}$ can be the probability density function of a continuous random variable.

2. What do you mean by MGF? Why it is called so?

3. The joint pdf of two dimensional random variables (X,Y) is given by $f(x,y)=\begin{cases}Kxe^{-y} & ; \ 0<x<2, y>0 \\ 0 & ; \quad \text{otherwise}\end{cases}$.

   Find the value of K.

4. Comment on the statement : "If $COV(X,Y)=0$, then X and Y are uncorrelated".

5. A radioactive source emits particles at a rate of 5 per min in accordance with Poisson process. Each particle emitted has a probability 0.6 of being recorded. Find the probability that 10 particles are recorded in 4 min period.

6. Check whether the Markov chain with transition probability matrix $P=\begin{pmatrix}0 & 1 & 0 \\ 1/2 & 0 & 1/2 \\ 0 & 1 & 0\end{pmatrix}$ is irreducible or not?

7. State the relationship between expected number of customers in the queue and in the system.

8. What is the steady state condition for M/M/C queueing model?

9. State Jackson's theorem for an open network.

10. What do you mean by level of multiprogramming in closed queueing network?

PART B — (5 × 16 = 80 marks)

11. (a) (i) If $f(x)=\begin{cases} xe^{-x^2/2} & ; \quad x \geq 0 \\ 0 & ; \quad x > 0 \end{cases}$, then show that $f(x)$ is a pdf and find $F(x)$. (8)

(ii) Find the MGF of a Poisson random variable and hence find its mean and variance. (8)

Or

(b) (i) A random variable X takes the values –2, –1, 0 and 1 with probabilities $\frac{1}{8},\frac{1}{8},\frac{1}{4}$ and $\frac{1}{2}$ respectively. Find and draw the probability distribution function. (8)

(ii) In a normal distribution, 31% of the items are under 45 and 8% are over 64. Find the mean and variance of the distribution. (8)

12. (a) The following table represents the joint pmf of two dimensional random variable (X,Y): (16)

(i) Evaluate marginal distribution of X and Y

(ii) Find the conditional distribution of X given $y=2$

(iii) Find the conditional distribution of Y given $x=3$

(iv) Find $P(X \leq 2,\ Y = 3)$

(v) Find $P(X+Y<4)$.

Or

(b) (i) Calculate the coefficient of correlation for the following data : (8)

X:	9	8	7	6	5	4	3	2	1
Y:	15	16	14	13	11	12	10	8	9

(ii) If $X_1, X_2, \cdots X_n$ are Poisson variates with parameters $\lambda = 2$, use the central limit theorem to estimate $P(120 \leq S_n \leq 160)$, where $S_n = X_1 + X_2 + \cdots + X_n$ and $n = 75$. (8)

13. (a) A soft water plant works properly most of the time. After a day in which the plant is working, the plant is working the next day with probability 0.95. Otherwise a day or repair followed by a day of testing is required to restore the plant to working status. Draw the state transition diagram for the status of the plant. Write down the tpm and classify the status of the process.

Or

(b) (i) Suppose that children are born at a Poisson rate of five per day in a certain hospital. What is the probability that (1) atleast two babies are born during the next six hours, (2) no babies are born during the next two days?

(ii) An engineer analyzing a series of digital signals generated by a testing system observes that only 1 out of 15 highly distorted signals follows a highly distorted signal, with no recognizable signals between, whereas 20 out of 23 recognizable signals follow recognizable signals, with no highly distorted signals between. Given that only highly distorted signals are not recognizable, find the fraction of signals that are highly distorted.

14. (a) The local one–person barber shop can accomodate a maximum of 5 people at a time (4 waiting and 1 getting hair–cut). Customers arrive according to a Poisson distribution with mean 5 per hour. The barber cuts hair at an average rate of 4/hr (exponential service time)

(i) What percentage of time is the barber idle?

(ii) What fraction of the potential customers are turned away?

(iii) What is the expected number of customers waiting for a hair–cut?

(iv) How much time can a customer expect to spend in the barber shop?

Or

(b) If people arrive to purchase cinema tickets at the average rate of 6/min, it takes an average of 7.5 seconds to purchase a ticket. If a person arrives 2min before the picture starts and if its takes exactly 1.5 min to reach the correct seat after purchasing the tickets,

(i) Can he expect to be seated for the start of the picture?

(ii) What is the probability that he will be seated for the start of the picture?

(iii) How early must he arrive in order to be 99% sure of being seated for the start of the picture?

15. (a) Derive Pollaczek–Khintchine formula for M/G/1 queueing model.

Or

(b) Write a brief note on the following :

(i) Open queueing network

(ii) Closed queueing network.

———————