Reg. No. :

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B.E./B.Tech. DEGREE EXAMINATION, NOVEMBER/DECEMBER 2013.

Fourth Semester

Electronics and Communication Engineering

MA 2261/MA 45/MA 1253/10177 PR 401/080380009 — PROBABILITY AND RANDOM PROCESSES

(Common to Biomedical Engineering)

(Regulation 2008/2010)

Time : Three hours Maximum : 100 marks

Use of statistical tables is permitted.

Answer ALL questions.

PART A — (10 × 2 = 20 marks)

1. Define Random Variable.

2. Define geometric distribution.

3. The joint pdf of the RV (x, y) is given by $f(x, y) = kxy\,e^{-(x^2+y^2)}$; $x > 0$, $y > 0$. Find the value of k.

4. Given the RV X with density function

$$f(x) = \begin{cases} 2x, & 0 < x < 1 \\ 0, & \text{elsewhere} \end{cases}$$

Find the pdf of $y = 8x^3$.

5. Define random process.

6. Define Markov process.

7. Define power spectral density function.

8. State Wiener-Khinchine theorem.

9. Define white noise process.

10. Define linear time invariant system.

PART B — ($5 \times 16 = 80$ marks)

11. (a) (i) Derive Poisson distribution from binomial distribution.

(ii) Find mean and variance of Gamma distribution.

Or

(b) (i) Suppose that a customers arrive at a bank according to a Poisson process with a mean rate of 3 per minute. Find the probability that during a time interval of 2 min.

(1) exactly 4 customers arrive and

(2) more than 4 customers arrive

(ii) If X and Y are independent RVs each normally distributed with mean zero and variance σ^2, find the pdf of $R=\sqrt{X^2+Y^2}$ and $\phi=\tan^{-1}\left(\frac{Y}{X}\right)$.

12. (a) (i) State and prove central limit theorem for iid RVs.

(ii) If X and Y are independent RVs with pdf's e^{-x}; $x \geq 0$ and e^{-y}; $y \geq 0$, respectively, find the pdfs of $U=\frac{X}{X+Y}$ and $V=X+Y$. Are U and V independent?

Or

(b) The joint probability mass function of (X, Y) is given by $p(x, y)=k(2x+3y)$, $x=0, 1, 2$; $y=1, 2, 3$. Find all the marginal and conditional probability distributions. Also find the probability distribution of $(X+Y)$.

13. (a) (i) If the two RVs A_r and B_r are uncorrelated with zero mean and $E(A_r^2)=E(B_r^2)=\sigma_r^2$, show that the process

$$x(t)=\sum_{r=1}^{n}(A_r \cos w_r t + B_r \sin w_r t) \text{ is wide-sense stationary.}$$

(ii) If $\{x(t)\}$ is a Gaussian process with $\mu(t)=10$ and $C(t_1, t_2)=16e^{-|t_1-t_2|}$, find the probability that (1) $X(10) \leq 8$ and (2) $|X(10)-X(6)| \leq 4$.

Or

(b) (i) Define Random telegraph signal process and prove that it is wide-sense stationary.

(ii) Prove that sum of two independent Poisson processes is a Poisson process.

14. (a) (i) The autocorrelation function of the random telegraph signal process is given by $R(\tau) = a^2 e^{-2\sqrt{|\tau|}}$. Determine the power density spectrum of the random telegraph signal.

(ii) The autocorrelation function of the Poisson increment process is given by

$$R(\tau) = \begin{cases} \lambda^2 & \text{for } |\tau| > \in \\ \lambda^2 + \dfrac{\lambda}{\in}\left(1 - \dfrac{|\tau|}{\in}\right) & \text{for } |\tau| \le \in \end{cases}$$

Prove that its spectral density is

$$S(w) = 2\pi\lambda^2 \delta(w) + \frac{4\lambda \sin^2\left(w \in /2\right)}{\in^2 w^2}.$$

Or

(b) (i) If the power spectral density of a WSS process is given by

$$S(w) = \begin{cases} \dfrac{b}{a}(a - |w|), & |w| \le a \\ 0, & |w| > a \end{cases}$$

Find the autocorrelation function of the process.

(ii) If the process $\{X(t)\}$ is defined as $X(t) = Y(t)Z(t)$ where $\{Y(t)\}$ and $\{Z(t)\}$ are independent WSS processes, prove that

(1) $R_{xx}(\tau) = R_{yy}(\tau) R_{zz}(\tau)$ and

(2) $S_{xx}(w) = \dfrac{1}{2\pi}\displaystyle\int_{-\infty}^{\infty} S_{yy}(\alpha) S_{zz}(w - \alpha)\, d\alpha$.

15. (a) (i) If $Y(t) = A\cos(w_0 t + \theta) + N(t)$, where A is a constant, θ is a random variable with a uniform distribution in $(-\pi, \pi)$ and $\{N(t)\}$ is a band-limited Gaussian white noise with a power spectral density

$$S_{NN}(w) = \begin{cases} \dfrac{N_0}{2}, & \text{for } |w - w_0| < w_B \\ 0, & \text{elsewhere} \end{cases}$$

Find the power spectral density of $\{Y(t)\}$. Assume that $N(t)$ and θ are independent.

(ii) Prove that the spectral density of any WSS process is non-negative.

Or

(b) $X(t)$ is the input voltage to a circuit (system) and $Y(t)$ is the output voltage. $\{X(t)\}$ is a stationary random process with $\mu_x = 0$ and $R_{xx}(\tau) = e^{-\alpha|\tau|}$. Find μ_y, $S_{yy}(w)$ and $R_{yy}(\tau)$, if the power transfer function is $H(w) = \dfrac{R}{R + i\,Lw}$

$$Y(t) = \int_{-\infty}^{\infty} h(\alpha) X(t-\alpha)\, d\alpha .$$
