Reg. No. : | | | | | | | | | | | | |

Question Paper Code : 91580

B.E./B.Tech. DEGREE EXAMINATION, NOVEMBER/DECEMBER 2014.

Fourth Semester

Electronics and Communication Engineering

MA 2261/MA 45 /MA 1253/ 080380009/ 10177 PR 401 — PROBABILITY AND RANDOM PROCESSES

(Common to Biomedical Engineering)

(Regulation 2008/2010)

Time : Three hours | Maximum : 100 marks

(Use of statistical tables is permitted)

Answer ALL questions.

PART A — (10 × 2 = 20 marks)

1. Find c, if a continuous random variable X has the density function $f(x)=\frac{c}{1+x^2}, -\infty < x < \infty$.

2. Find the moment generating function of Poisson distribution.

3. Given the random variable X with density function $f(x)=\begin{cases} 2x, & 0<x<1 \\ 0, & elsewhere \end{cases}$. Find the pdf of $Y=8X^3$.

4. Define the joint pmf of a two-dimensional discrete random variable.

5. Define stochastic processes.

6. Define Markov process.

7. Write any two properties of autocorrelation.

8. Write the Wiener-Khintchine relation.

9. Define white noise.

10. The autocorrelation function for a stationary ergodic process with no periodic component is $R_{xx}(\tau)=25+\frac{4}{1+6\tau^2}$. Find the mean and variance of the process $\{X(t)\}$.

PART B — (5 × 16 = 80 marks)

11. (a) (i) Find the nth moment about mean of normal distribution.

(ii) Derive Poisson distribution from the binomial distribution.

Or

(b) (i) Find the mean and variance of Gamma distribution.

(ii) A random variable X has the pdf $f(x)=\begin{cases} 2e^{-2x}, & x \geq 0 \\ 0, & x < 0 \end{cases}$. Obtain the mgf and first four moments about the origin. Find mean and variance of the same.

12. (a) The joint probability mass function of (X, Y) is given by $p(x, y)=k(2x+3y)$, $x=0,1,2;\ y=1,2,3$. Find K and all the marginal and conditional probability distributions. Also find the probability distribution of $(X+Y)$

Or

(b) (i) State and prove central limit theorem.

(ii) The lifetime of a certain brand of an electric bulb may be considered a RV with mean 1200h and standard deviation 250h. Find the probability, using central limit theorem that the average lifetime of 60 bulbs exceed 1250h.

13. (a) (i) The process $\{X(t)\}$ whose probability distribution under certain condition is given by

$$P\{X(t)=n\} = \begin{cases} \dfrac{(at)^{n-1}}{(1+at)^{n+1}}; n=1,2,..... \\ \dfrac{at}{1+at}; n=0 \end{cases}$$

Show that it is not stationary.

(ii) If the 2n random variables A_r and B_r are uncorrelated with zero mean and $E(A_r^2)=E(B_r^2)=\sigma_r^2$, show that the process. $X(t)=\sum_{r=1}^{n}(A_r \cos w_r t + B_r \sin w_r t)$ is wide sense stationary. What are the mean and autocorrelation of $X(t)$?

Or

(b) (i) Define semi-random telegraph signal process and random telegraph signal process and prove also that the former is evolutionary and the latter is wide-sense stationary.

(ii) If $\{X(t)\}$ is a Gaussian process with $\mu(t)=10$ and $c(t_1, t_2)=16e^{-|t_1-t_2|}$ find the probability that (1) $X(10) \leq 8$ and (2) $|X(10)-X(6)| \leq 4$

14. (a) (i) The random binary transmission process $\{X(t)\}$ is a WSS process with zero mean and autocorrelation function $R(\tau)=1-\frac{|\tau|}{T}$, where T is a constant. Find the mean and variance of the time average of $\{X(t)\}$ over $(0,T)$. Is $\{X(t)\}$ mean ergodic?

(ii) Find the power spectral density of a WSS process with autocorrelation function $R(\tau)=e^{-\alpha\tau^2}$.

Or

(b) (i) A random process $\{X(t)\}$ is given by $X(t)=A\cos pt+B\sin pt$, where A and B are independent random variables such that $E(A)=E(B)=0$ and $E(A^2)=E(B^2)=\sigma^2$. Find the power spectral density of the process.

(ii) If the power spectral density of a WSS process is given by $S(w)=\begin{cases}\frac{b}{a}(a-|w|), & |w|\le a\\ 0, & |w|>a\end{cases}$, find the autocorrelation function of the process.

15. (a) (i) Check whether the following systems are linear (1) $y(t)=t\,x(t)$ (2) $y(t)=x^2(t)$.

(ii) The power spectral density of a signal $x(t)$ is $S_x(w)$ and its power is P. Find the power of the signal $bx(t)$.

Or

(b) A linear system is described by the impluse response $h(t)=\frac{1}{Rc}e^{-\left(\frac{t}{Rc}\right)}$. Assume an input signal whose autocorrelation function is $B\delta(\tau)$. Find the autocorrelation mean and power of the output.