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Question Paper Code : 73770

B.E./B.Tech. DEGREE EXAMINATION, APRIL/MAY 2017.

Fourth Semester

Computer Science and Engineering

MA 2262/MA 44/MA 1252/10177 PQ 401/080250008 — PROBABILITY AND QUEUEING THEORY

(Common to Information Technology)

(Regulations 2008/2010)

Time : Three hours Maximum : 100 marks

Statistical Tables may be permitted.

Answer ALL questions.

PART A — (10 × 2 = 20 marks)

1. A continuous random variable X that can assume any value between $x = 2$ and $x = 5$ has a density function given by $f(x) = k(1 + x)$. Find $P(X < 4)$.

2. Identify the random variable and name the distribution it follow from the following statement :

   "A realtor claims that only 30% of the houses in a certain neighbourhood, are appraised at less than ₹ 20 lakhs. A random sample of 10 houses from that neighbourhood is selected and appraised. To check the realtor's claim is acceptable are not".

3. Determine the value of the constant c if the joint density function of two discrete random variables X and Y is given by $p(m, n) = cmn$, $m = 1, 2, 3$ and $n = 1, 2, 3$.

4. The lines of regression in a bivariate distribution are $X + 9Y = 7$ and $Y + 4X = \frac{49}{3}$. Find the coefficient of correlation between X and Y.

5. Define wide sense stationary process.

6. If the transition probability matrix (tpm) of a Markov chain is $\begin{pmatrix} 0 & 1 \\ \frac{1}{2} & \frac{1}{2} \end{pmatrix}$, find the steady state distribution of the chain.

7. Define Markovian Queueing Models.

8. Suppose that customers arrive at a Poisson rate of one per every 12 minutes and that the service time is exponential at a rate of one service per 8 minutes.

   (a) What is the average number of customers in the system?

   (b) What is the average time of a customer spends in the system?

9. State Jackson's theorem for an open network.

10. What do the letter in the symbolic representation M/G/1 of a queueing model represent?

PART B — (5 × 16 = 80 marks)

11. (a) (i) If the random variable X takes the values 1, 2, 3 and 4 such that $2P(X=1)=3P(X=2)=P(X=3)=5P(X=4)$, then find the probability distribution and cumulative distribution function of X. (8)

(ii) Find the MGF of the binomial distribution and hence find its mean. (8)

Or

(b) (i) If the probability that an applicant for a driver's licence will pass the road test on any given trial is 0.8, what is the probability that he will finally pass the test (1) on the 4th trial (2) in fewer than 4 trials? (8)

(ii) The number of monthly breakdowns of a computer is a random variable having a Poisson distribution with mean equal to 1.8. Find the probability that this computer will function for a month (1) without a breakdown (2) with only one breakdown. (8)

12. (a) (i) The probability density function of X and Y is given by

$$f(x, y) = \frac{6}{7}\left(x^2 + \frac{xy}{2}\right), \ 0 < x < 1, \ 0 < y < 2.$$

(1) Compute the marginal density function of X and Y.

(2) Find $E(X)$ and $E(Y)$

(3) Find $P\left(X < \frac{1}{2}, Y > \frac{1}{2}\right)$. (10)

(ii) If X and Y each follow an exponential distribution with parameter 1 and they are independent, find the probability density function of $U = X - Y$. (6)

Or

(b) (i) If X, Y and Z are uncorrelated random variables with zero means and standard deviation 5, 12 and 9 respectively and if $U = X + Y$ and $V = Y + Z$, find the correlation coefficient between U and V. (10)

(ii) If $X_1, X_2, ..., X_n$ are Poisson variates with parameter $\lambda = 2$, use central limit theorem to estimate $P(120 < S_n < 160)$, where $S_n = X_1 + X_2 + ... + X_n$ and $n = 75$. (6)

13. (a) (i) Prove that the process $\{X(t)\}$ whose probability distribution given by $P[X(t) = n] = \begin{cases} \dfrac{(at)^{n-1}}{(1+at)^{n+1}}, & n = 1, 2, 3, ... \\ \dfrac{at}{1+at}, & n = 0 \end{cases}$ is not stationary. (8)

(ii) The TPM of a Markov chain $\{X_n\}$, $n = 1, 2, 3, ...$ having three states 1, 2 and 3 is $P = \begin{bmatrix} 0.1 & 0.5 & 0.4 \\ 0.6 & 0.2 & 0.2 \\ 0.3 & 0.4 & 0.3 \end{bmatrix}$ and the initial distribution is $P^{(0)} = \{0.7, 0.2, 0.1\}$. Find

(1) $P[X_2 = 3]$,

(2) $P[X_3 = 2, X_2 = 3, X_1 = 3, X_0 = 2]$. (8)

Or

(b) (i) A salesman territory consists of three cities A, B and C. He never sells in the same city on successive days. If he sells in city A, then the next day he sells in city B. However, if he sells in either B or C the next day he is twice as likely as to sell in city A as in the other city. In the long run how often does he sell in each the cities? (8)

(ii) Suppose that customers arrive at a bank according to a Poisson process with mean rate of 3 per minute. Find the probability that during a time interval of two minutes

(1) Exactly 4 customers arrive

(2) More than 4 customers arrive

(3) Less than 4 customers arrive. (8)

14. (a) (i) Derive (1) L_s, average number of customers in the system (2) L_q, average number of customers in the queue for the queuing model (M/M/1): (N/FIFO). (8)

(ii) There are three typists in an office. Each typist can type an average of 6 letters per hour. If letters arrive for being typed at the rate of 15 letters per hour, what fraction of time all the typists will be busy? What is the average number of letters waiting to be typed? (Assume Poisson arrivals and exponential service times). (8)

Or

(b) Customers arrive at a one man barber shop according to a Poisson process with a mean inter arrival time of 20 minutes. Customers spend an average of 15 minutes in the barber chair. The service time is exponentially distributed. If an hour is used as a unit of time, then

(i) What is the probability that a customer need not wait for a hair cut?

(ii) What is the expected number of customer in the barber shop and in the queue?

(iii) How much time can a customer expect to spend in the barber shop?

(iv) Find the average time that a customer spend in the queue.

(v) Estimate the fraction of the day that the customer will be idle.

(vi) What is the probability that there will be 6 or more customers?

(vii) Estimate the percentage of customers who have to wait prior to getting into the barber's chair. (16)

15. (a) Derive Pollaczek-Khintchine formula of an $M/G/1$ Queueing model. (16)

Or

(b) (i) Write a brief note on the open queueing networks. (8)

(ii) A repair facility shared by a large number of machine has 2 series stations with respective service rates of 2 per hour and 3 per hour. If the average rate of arrival is 1 per hour, find

(1) The average number of machines in the system

(2) The average waiting time in the system.

(3) Probability that both service stations are idle. (8)

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