ANNA UNIVERSITY COIMBATORE

B.E. / B.TECH. DEGREE EXAMINATIONS : DECEMBER 2009

REGULATIONS - 2007

FOURTH SEMESTER - ELECTRONICS AND COMMUNICATION ENGINEERING

070030011 - PROBABILITY THEORYAND RANDOM PROCESS

TIME : 3 Hours Max.Marks : 100

PART – A

(20 x 2 = 40 MARKS)

ANSWER ALL QUESTIONS

1. If A,B,and C are any three events such that $P(A) = P(B) = P(C)=1/4, P(A \cap B) = P(B \cap C) = 0$, $P(C \cap A) = 1/8$. Find the probability that atleast one of the events A,B or C occurs.

2. A continuous random variable X has a pdf $f(x) = kx^2e^{-x}$; $x \geq 0$. Find k and the mean.

3. Find the moment generating function for the distribution

$$f(x) = \begin{cases} 2/3 & \text{at } x = 1 \\ 1/3 & \text{at } x = 2 \\ 0 & \text{otherwise} \end{cases}$$

4. The joint pdf of X and Y is given by

$$f(x,y) = \begin{cases} x^3y^3/16 & 0 \leq x \leq 2, 0 \leq y \leq 2 \\ 0 & \text{otherwise.} \end{cases}$$

Find $f_X(x)$.

5. The number of monthly breakdowns of a computer is a random variable having a Poisson distribution with mean equal to 1.8. Find the probability that this computer will function for a month without a breakdown.

6. Define the uniform distribution of a continuous random variable X.

7. The time (in hours) required to repair a machine is exponentially distributed with parameter $\lambda = ½$. What is the probability that the repair time exceeds 2 hours?

8. Write the mean and variance of the binomial distribution.

9. What are the four classifications of a random process?

10. Examine whether the Poison process {X(t)}, given by the probability law $P[X(t)=r] = e^{-\lambda t}(\lambda t)^r / \lfloor r$, {r=0,1,2,...} is covariance stationary.

11. In the fair coin experiment, we define the process {X(t)} as follows.

$$X(t) = \begin{cases} \sin \pi t, & \text{if head appears} \\ 2t & \text{if tail appears.} \end{cases}$$

Find E{X(t)}.

12. If {X(t)} is a wide-sense stationary process with autocorrelation $R(\tau) = Ae^{-\alpha|\tau|}$, determine the second order moment of the random variable X(8) - X(5).

13. Define autocorrelation.

14. Given that the autocorrelation function for a stationary ergodic process with no periodic components is $R_{XX}(\tau) = 25 + (4/(1+ 6\tau^2))$. Find the mean value and variance of the process {X(t)}.

15. State Wiener-Khinchine theorem.

16. Find the power spectral density of the random telegraph signal with auto correlation function $R(\tau)=a^2e^{-2\gamma|\tau|}$

17. Define Gaussian process.

18. Write a note on noise in communication system.

19. What is half wave linear detector process?

20. Name the commonly used filters in electrical systems and define them mathematically.

PART – B

(5 x 12 = 60 MARKS)

ANSWER ANY FIVE QUESTIONS

21. The joint pdf of a two dimensional random variable (X,Y) is given by $f(x,y) = xy^2+x^2/8,\ 0\le x\le 2, 0\le y\le 1$. Compute $P(X>1), P(Y<½), P(X>1/Y<1/2)$ and $P(X+Y\le 1)$.

22. a) If X is a random variable with probability distribution 6

X :	1	2	3
p(X) :	1/6	1/3	½

Find its mean ,variance and $E(4X^3+3X+11)$

b) Find the mean and variance of the geometric distribution. 6

23. State and prove the memory less property of the exponential distribution

24. Given a random variable with characteristic function $\phi(\omega) = E\{e^{i\omega Y}\}$ and a random process defined by $X(t) = \cos(\lambda t+Y)$, show that $\{X(t)\}$ is stationary in the wide sense.

25. a) Two random process X(t) and Y(t) are defined by $X(t) = A\cos\omega_0 t + B\sin\omega_0 t$ and $Y(t) = B\cos\omega_0 t - A\sin\omega_0 t$. Show that X(t) and Y(t) are jointly wide-sense stationary, if A and B are uncorrelated random variables with zero means and same variances, where ω_0 is a constant. 6

b) Define cross-correlation function and state any four of its properties. 6

26. State Mean-Ergodic process. Also state and prove Mean-Ergodic theorem

27. a) The power spectral density function of a zero mean WSS process $\{X(t)\}$ is given by 6

$$S(\omega) = \begin{cases} 1 & \text{if } |\omega|<\omega_0 \\ 0 & \text{elsewhere} \end{cases}$$

Find $R(\tau)$ and show also that $X(t)$ and $X(t+\tau/\omega_0)$ are uncorrelated.

b) Define (a)Thermal noise (b)White noise. 6

28. If $\{X(t)\}$ is a Gaussian process with $\mu(t) = 10$ and $C(t_1,t_2)=16e^{-|t_1-t_2|}$ find the probability that (i) $X(10)\le 8$ and (ii) $|X(10) - X(6)|\le 4$.

*****THE END*****