ANNA UNIVERSITY COIMBATORE

B.E. / B.TECH. DEGREE EXAMINATIONS : JUNE 2009

REGULATIONS - 2007

FOURTH SEMESTER : ECE BRANCH

070030011 - PROBABILITY THEORY AND RANDOM PROCESS

TIME : 3 Hours Max.Marks : 100

PART – A

(20 x 2 = 40 MARKS)

ANSWER ALL QUESTIONS

1. The M.C.A. class consist of 60 students, In that 5 of them are girls, 20 of them are rich, 10 them are fair complexioned. What is the probability of selecting a fair complexioned rich girl?

2. Verify whether $f(x) = \begin{cases} |x|, & \text{if } -1 \le x \le 1 \\ 0, & \text{other wise} \end{cases}$
can be the p.d.f of a continuous random variable

3. If the M.G.F of a random variable is $\frac{2}{2-t}$. Find the S.D of X

4. If $f(x,y) = 4xy\, e^{-(x^2+y^2)}$ for x,y ≥ 0. Examine if x,y are independent

5. Find the M.G.F. of the Geometric Distribution $P(X=x) = q^{x-1} p$, x = 1,2,3,4.....

6. The cumulative distribution function of a random variable X is given by

$$F(x) = \begin{cases} 0 & \text{if } x \le 0 \\ \frac{x}{2\pi} & \text{if } 0 < x \le 2\pi \\ 1 & \text{if } x > 2\pi \end{cases}$$

Find the p.d.f. of X and show that X has a uniform distribution in the interval (0, 2 π). Find also $P(\pi/4 \le X \le \pi/2)$

7. If X is poisson variate such that P(X=0) = 0.5, Find Var (X)

8. The mean and variance of a binomial distribution are 4 and 4/3 respectively. Find $P(X \ge 1)$, if n = 6.

9. What is the difference between a random variable and random process?

10. Define wide – sense stationary process.

11. Define a Gaussian process

12. Prove that the Poisson process is a Markov process.

13. If $R(\tau)$ is the Auto correlation function prove that $R(\tau) = R(-\tau)$

14. Find the variance of the stationary process (X(t)), whose auto correlation function is given by

$$R(\tau) = 16 + \frac{9}{1+6\tau^2}$$

15. Define the cross – correlation function

16. Define the power spectral density function of a stationary process

17. Find the mean – square value (or the average power) of the process (X(t)) if its Auto correlation function is given by $R(\tau) = e^{-\tau^2/2}$

18. State Wiener – Khinchine theorem

19. Find the power spectral density function of a stationary process whose auto correlation function is $e^{-|\tau|}$

20. The Auto correlation function of the white noise is given by $R_{NN}(\tau) = \frac{N_0}{2} E(t)$. Find spectral density of white noise.

PART – B

(5 X 12 = 60 MARKS)

ANSWER ANY FIVE QUESTIONS

21. (a) A continuous random variable X has p.d.f.
$f(x) = k x^2 e^{-x}$, x >0, find k, mean and variance. (6)

21. (b) A random variable X has the following probability distribution.

x	0	1	2	3	4	5	6	7	8
P(x)	a	3a	5a	7a	9a	11a	13a	15a	17a

i. Find a,

ii. Find the distribution function of X

iii. What is the smallest value of x for which $P(X \le x) > 0.5$ (6)

22. (a) Find the M.G.F. of the Exponential distribution and deduce its mean and variance. (6)

(b) Prove that all the odd ordered central moments about mean of a normal distribution vanish. (6)

23. (a) Define Gamma distribution and find its M.G.F. (6)

(b) The daily consumption of milk in the city in excess of 20,000 gallons, is approximately distributed as a gamma variate with the parameters k = 2 and λ = 1/10,000. The city has a daily stock of 30,000 gallons. What is the probability that the stock is insufficient on a particular day. (6)

24. (a) Two random variables X and Y have the following joint p.d.f. $f(x,y) = k(4-x-y)$ for $0 \le x \le 2$, $0 \le y \le 2$. Find k, Marginal distributions of X and Y and conditional distribution of Y given X. (6)

(b) Suppose that customers arrive at a bank according to a poisson process with a mean rate of 3 per minute. Find the probability that during a time interval of 2 min

i. exactly 4 customers arrive

ii. More than 4 customers arrive (6)

25. a) Three boys A, B, and C are throwing a ball to each other. A always throws the ball to B and B always throws the ball to C, but C is just as likely to throw the ball to B as to A. Show that the process is Markovian. Find the transition matrix and classify the states. (6)

(b) The auto correlation function of the random telegraph signal process is given by $R(\tau) = \alpha^2 e^{-2\gamma|\tau|}$

Determine the power density spectrum of the random telegraph signal. (6)

26. (a) Show that the process (X(t)), $X(t) = A\cos(\omega t + \theta)$ where A and ω are constants. Θ is uniformly distributed in $[-\pi, \pi]$ is wide sense stationary (6)

(b) If (X(t)) and (Y(t)) are two random process then show that $|R_{xy}(\tau)| \le \sqrt{R_{xx}(0)R_{yy}(0)}$ where $R_{xx}(\tau)$ and $R_{yy}(\tau)$ are their respective auto correlation functions. (6)

27. (a) If the power spectral density of a WSS process is given by

$$S(\omega) = \begin{cases} \frac{b}{a}(a-|\omega|), & \text{if } |\omega| \le a \\ 0 & \text{if } |\omega| > a \end{cases}$$

Find the auto correlation function of the process (6)

(b) Find the power spectral density of a WSS process with auto correlation. Function. $R(\tau) = e^{-\alpha\tau^2}$ (6)

28. (a) Prove that if the input to a time – invariant stable linear system is a WSS process, the output will also be a WSS process. (6)