ANNA UNIVERSITY COIMBATORE

B.E. / B.TECH. DEGREE EXAMINATIONS : MAY / JUNE 2010

REGULATIONS : 2007

FOURTH SEMESTER : ECE

070030011 - PROBABILITY THEORY AND RANDOM PROCESS

TIME : 3 Hours Max.Marks : 100

PART – A

(20 x 2 = 40 MARKS)

ANSWER ALL QUESTIONS

1. Define probability density function and mention its properties.
2. Find the expected value of a continuous random variable X whose PDF is given by $f_X(x) = 2e^{-2x}, \quad x \geq 0$.
3. Find M.G.F of Poisson disfribution.
4. State the properties of Normal distribution.
5. The joint PDF of two random variables X and Y is given by $f_{XY}(x, y)$. If we define the random variable $U = XY$, determine the PDF of U.
6. The joint PMF of two discrete random variables X and Y is given by $P_{XY}(x, y) = 2xy, \quad x = 1,2,3 \quad y = 1,2,3$
   $= 0 \quad otherwise$, find the marginal PMFs of X.
7. Write the rule for two random variables X and Y are independent.
8. State central limit theorem.
9. Define deterministic and nondeterministic processes with an example.
10. What is a first order stationary process?
11. Define Wide Sense Stationary process with an example.
12. If customers arrive at a counter in accordance with a Poisson process with a mean rate of 2 per minute, find the probability that the interval between 2 consecutive arrivals is (i) more that 1 min, (ii) between 1 min and 2 min.
13. Find the variance of the stationary process $\{X(t)\}$ whose auto correlation function is given by $R(\tau) = 25 + \frac{4}{1+6\tau^2}$.
14. Write the properties of power spectral density function.
15. Find the cross correlation function of a cross-power spectrum defined by $S_{XY}(w) = \begin{cases} a + jb\omega/W, & -W < \omega < W \\ 0 & elsewhere \end{cases}$ where $W > 0$, a and b are real constants.
16. Express the Wiener-Khintchine relation.
17. Prove that $R_{XY}(\tau)$ is an even function of τ.
18. Define linear system with an example.
19. The stages in a three-stage amplifier have effective input noise temperatures $T_{e1} = 1350K$, $T_{e2} = 1700K$ and $T_{e3} = 2600K$. The respective available power gains are $G_1 = 16$, $G_2 = 10$ and $G_3 = 6$. Find effective input noise Temperatures of the overall amplifier.
20. Define impulse function.

PART – B

(5 x 12 = 60 MARKS)

ANSWER ANY FIVE QUESTIONS

21. a. Derive the mean and variance of uniform distribution in the interval $[a, b]$ and also find the mean and variance of the time that Joe takes to grade a paper, the time that he, the teaching assistant, takes to grade a paper is uniformly distributed between 5 minutes and 10 minutes. 6

21. b. A continuous random variable X has the p.d.f $f(x)=\begin{cases}\frac{1}{2}(x+1), & -1<x<1\\ 0 & otherwise\end{cases}$ find the mean and variance. 6

22. a. The weights of parcels that are dropped off at a local shipping center can be represented by a random variable X that is normally distributed with mean $\mu_X = 70$ and standard deviation $\sigma_X = 10$. Determine the following: 6

i. $P(X) > 50$, ii. $P(X) < 60$, iii. $P(60 < X < 90)$

b. Find the coefficient of correlation between industrial production and export using the following data: 6

Production (X)	55	56	58	59	60	60	62
Expert (Y)	35	38	37	39	44	43	44

23. From the following data, find

(i) The two regression equations

(ii) The coefficient of correlation between the marks in economics and statistics

(iii) The most likely marks in statistics when marks in economics are 30.

Marks in Economics	25	28	35	32	31	36	29	38	34	32
Marks in Statistics	43	46	49	41	36	32	31	30	33	39

24. a. The joint PDF of the random variables X and Y is defined as follows. 6

$f_{XY}(x,y) = 25e^{-5y}, \quad 0 < x < 0.2, y > 0.$

$= 0$ *otherwise*

i. Find the marginal PDFs of X and Y.

ii. What is the covariance of X and Y?

b. A fair dice is tossed repeatedly. If X_n denotes the maximum of the numbers occurring in the first n tosses, find the transition probability matrix P of the Markov chain $\{X_n\}$. Also find P^2 and $P(X_2 = 6)$. 6

25. a. A random process $\{X(t)\}$ is defined by $X(t) = A\cos t + B\sin t, -\infty < t < \infty$ where A and B are independent random variables each of which has a value −2 with probability 1/3 and a value 1 with probability 2/3. Show that $\{X(t)\}$ is a wide sense stationary process. 6

b. Prove that the random process $X(t) = \cos(\omega t + \theta)$ is an ergodic process, where ω is a constant and θ is a random variable with probability density 6

$$f(\theta) = \begin{cases}\frac{1}{2\pi}, & 0 \le \theta \le 2\pi\\ 0, & otherwise\end{cases}.$$

26. a. Two random processes $X(t)$ and $Y(t)$ are defined as follows: 6

$X(t) = A\cos(wt+\theta), Y(t) = B\sin(wt+\theta)$ where A, B and w are constants and θ is a random variable that is uniformly distributed between 0 and 2π. Find the cross correlation function of $X(t)$ and $Y(t)$.

26. b. Express the autocorrelation function of the process $\{X'(t)\}$ in terms of the autocorrelation function of the process $\{X(t)\}$. 6

27. a. Prove that in a random process $|R_{XY}(\tau)| \le \sqrt{R_{XX}(0)\, R_{YY}(0)}$. 6

b. Explain briefly about the representation of noise, thermal noise and white noise. 6

28. a. Prove that if the input $X(t)$ and its output $Y(t)$ are related by $Y(t) = \int_{-\infty}^{\infty} h(u)\, X(t-u)\, du$, then the system is a linear time invariant system. 6

b. A random process $X(t)$ is the input to a linear system whose impulse response is $h(t) = 2e^{-t}$, $t \ge 0$. If the autocorrelation function of the process is $R_{XX}(\tau) = e^{-2|\tau|}$, find the power spectral density of the output process $Y(t)$. 6

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