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ANNA UNIVERSITY COIMBATORE

B.E. / B.TECH. DEGREE EXAMINATIONS : OCTOBER 2009

REGULATIONS – 2007

FOURTH SEMESTER : ELECTRONICS & COMMUNICATION ENGG.

070030011 - PROBABILITY THEORY AND RANDOM PROCESS

TIME : 3 Hours Max.Marks : 100

PART – A

(20 x 2 = 40 MARKS)

ANSWER ALL QUESTIONS

1. The probability that a contractor will get a plumbing contract is 2/3 and the probability that he will get an electric contract is 4/9. If the probability of getting atleast one contract is 4/5, what is the probability that he will get both?
2. The distribution function of a random variable X is given by F(x) = 1 – (1+x) λ^{-x}, x ≥ 0. Find the density function, mean and variance of X.
3. A continuous random variable X that can assume any value between x = 2 and x = 5 has density function given by f (x) = k (1 + x), find P(X < 4).
4. If the probability that an applicant for a driver's license will pass the road test on any given trial is 0.8, what is the probability that he will finally pass the test on the fourth trial?
5. Define exponential random variable. Give an example.
6. Define Poisson and negative binomial distributions.
7. Sharon and Ann play a series of backgammon games until one of them wins five games. Suppose that the games are independent and the probability that Sharon win a game is 0.58.

8. For $\lambda > 0$, let F(x,y)= $\begin{cases} 1-\lambda\lambda^{-\lambda(x+y)} \\ 0 \end{cases}$ if x > 0,y > 0 otherwise check whether F can be the joint probability distribution function of two random variables X and Y.
9. Define autocorrelation of a process X(t).
10. State two properties of cross correlations.
11. Show that the Poisson process X(t) given by P (X(t) = n) = $\dfrac{\lambda^{-\lambda t}(\lambda t)^r}{\angle r}$ is not covariance stationary.
12. Show that $cov^2(x, y) \le var(x)\, var(y)$.
13. Find the variance of the stationary process X(t) whose autocorrelation function given by R (z) = 16 + $\dfrac{9}{1+6z^2}$.
14. What is a Markov chain? When a markov chain is homogeneous.
15. What do you mean by transient state and steady state queuing system?
16. Consider the random process X(t) = cos(ω_0t + θ) where θ uniformly distributed in the interval - π to π. Check whether X(t) is stationary or not?
17. Consider the Markov chain with transition probability matrix:
$$\begin{bmatrix} 0.4 & 0.6 & 0 & 0 \\ 0.3 & 0.7 & 0 & 0 \\ 0.2 & 0.4 & 0.1 & 0.3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$
is it irreducible? If not, find the classes. Find the nature of states.
18. If *P(A)* = 0.4, *P(B)* = 0.7, prove that *P(A* ∩ *B)* ≥ 0.3, find *P*($\overline{A} \cap \overline{B}$).

19. Five men out of 100 and 25 women out of 1000 are colour-blind. A colour-blind person is chosen at random. What is the probability that the person is a male ? (Assume males and females are in equal number).

20. If the cdf of a RV X is given by f(x) =1 - $\lambda^{-\lambda x}$, when $x \geq 0$ and = 0,when $x < 0$,find the pdf of X.

PART – B

(5 x 12 = 60 MARKS)

ANSWER ANY FIVE QUESTIONS

21. a) Given the joint density function, f(x,y) = cx (x-y), $0 < x < 2$, $-x < y < x$. Evaluate c. Find the marginal densities of X and Y. Find the conditional density of Y given X= x. 8

b) For the following data find the most likely price at Madras corresponding to the price 70 at Bombay and that at Bombay corresponding to the price 68 at Madras: 4

	Madras	Bombay
Average price	65	67
S.D of price	0.5	3.5

S.D of the difference between the prices at Madras and Bombay is 3.1?

22. a) A box contains 2000 components of which 5% are defective. A second box contains 500 components 500 components of which 40% are defective. Two other boxes contains 1000 components, each with 10% defective components. We select at random one of the above boxes and remove from it at random a single component. 8

a) What is the probability that the component is defective?

b) Finding that the selected component is defective, what is the probability that it was drawn from box 2?

22. b) A random variable X has the following probability distribution. 4

x :	-2	-1	0	1	2	3
p(x):	0.1	K	0.3	2K	0.3	3K

Find K, (b) Evaluate P(X<2) and P(-2 < X < 2)

23. a) Find the mean and variance of Geometric distribution. 6

b) It is known that the probability of an item produced by a certain machine will be defective is 0.05. If the produced items are sent to the market in packets of 20, find the number of packets containing at least, exactly and at most 2 defective items in a consignment of 1000 packets using poisson approximation to binomial distribution. 6

24. a) The marks obtained by a number of students in a certain subject are approximately normally distributed with mean 65 and standard deviation 5. If 3 students are selected at random from this group, what is the probability that at least 1 of them would have scored above 75? 6

b) Buses arrive at a specified stop at 15 min. intervals starting at 7 A.M., that is, they arrive at 7, 7:15, 7:45 and so on. If a passenger arrives at the stop at a random time that is uniformly distributed between 7 and 7:30 A.M., find the probability that he waits 6

a) less than 5 min for a bus and

b) at least 12 min. for a bus

25. a) Show that the random process $X(t) = A \cos (\omega_0 t + \theta)$ is wide – sense stationary, if A and ω_0 are contants and θ is a uniformly distributed RV in $(0, 2\pi)$. 6