Reg. No. :

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B.E./B.Tech. DEGREE EXAMINATION, NOVEMBER/DECEMBER 2016.

Fourth Semester

Computer Science and Engineering

MA 2262/MA 44/MA 1252/080250008/10177 PQ 401 — PROBABILITY AND QUEUEING THEORY

(Common to Information Technology)

(Regulations 2008/2010)

Time : Three hours Maximum : 100 marks

Statistical Tables may be permitted.

Answer ALL questions.

PART A — (10 × 2 = 20 marks)

1. A random variable X takes the values 1, 2, 3, 4 such that $2P(X=1)=3P(X=2)=P(X=3)=5P(X=4)$. Find the probability distribution of X.

2. Write down the mean and variance of the Weibull distribution.

3. Given the two regression lines $x+2y-5=0$ and $2x+3y-8=0$. Find the mean values of x and y.

4. State central limit theorem.

5. What is a continuous random sequence? Give an example.

6. If the initial state probability distribution of a Markov chain is $p^{(0)}=\left(\frac{5}{6},\frac{1}{6}\right)$ and the tpm of the chain is $\begin{pmatrix} 0 & 1 \\ 1/2 & 1/2 \end{pmatrix}$, find the probability distribution of the chain after 2 steps.

7. What are the characteristics of a queuing system?

8. If $\lambda = 4$/hr and $\mu = 10$/hr in an (M/M/1): (3/FIFO) queuing system, find the probability that there is no customer in the system.

9. What do you mean by a series queue with blocking?

10. Define 'Bottle neck' of the system in queue networks.

PART B — ($5 \times 16 = 80$ marks)

11. (a) (i) If a random variable X has a cumulative distribution function $F(x) = \begin{cases} 0, & \text{for } x \le 0 \\ c(1 - e^{x}) & \text{for } x > 0 \end{cases}$, find the probability density function, the value of c and $P(1 < x < 2)$. (8)

(ii) Find the moment generating function and r^{th} moment for the distribution whose probability density function is $f(x) = e^{-x}$, $0 \le x \le \infty$. Also find the first three moments about mean. (8)

Or

(b) (i) It is known that the probability of an item produced by a certain machine will be defective is 0.05. If the produced items are sent to the market in packets of 20, find the number of packets containing at least, exactly and at most 2 defective items in a consignment of 1000 packets. (8)

(ii) State and prove memory less property of exponential distribution. Using this property, solve the following problem :

The length of the shower on a tropical island during rainy season has an exponential distribution with parameter 2, time being measured in minutes. If a shower has already lasted for 2 minutes, what is the probability that it will last for at least one more minute? (8)

12. (a) (i) The joint distribution of X and Y is given by $f(x, y) = \dfrac{x + y}{21}$ for $x = 1,2,3$ and $y = 1, 2$. Find the marginal distribution of X and Y, the conditional distribution of X given $Y = 1$ and the conditional distribution of Y given $X = 2$. (8)

(ii) The joint probability density function of a two dimensional random variable (X, Y) is given by $f(x, y) = \begin{cases} 2, & 0 < x < 1,\ 0 < y < x \\ 0, & \text{otherwise} \end{cases}$. Are X and Y independent? Also find the conditional density functions of X and Y. (8)

Or

(b) (i) The joint probability density function of a RV (X,Y) is $f(x,y)=\frac{1}{8}(6-x-y)$, $0<x<2$, $2<y<4$. Find the correlation coefficient between X and Y. (12)

(ii) X and Y are independent random variables with $f(x)=e^{-x}U(x)$ and $f(y)=3e^{-y}U(y)$. Find the joint probability density function of (Z,W) if $Z=\frac{X}{Y}$ and $W=Y$. (4)

13. (a) (i) Two random processes $X(t)$ and $Y(t)$ are defined by $X(t)=A\cos\lambda t+B\sin\lambda t$ and $Y(t)=B\cos\lambda t-A\sin\lambda t$. Show that $X(t)$ and $Y(t)$ are jointly wide sense stationary, if A and B are uncorrelated random variables with zero means and the same variances and λ is a constant. (12)

(ii) Prove that difference of two independent Poisson processes is not a Poisson process. (4)

Or

(b) (i) A fair dice is tossed repeatedly. If X_n denotes the maximum of the numbers occurring in the first n tosses, find the transition probability matrix P of the Markov chain $\{X(n)\}$. Also find $P\{X_2=6\}$. (8)

(ii) The three – state Markov chain is given by the transition probability matrix $P=\begin{pmatrix} 0 & 2/3 & 1/3 \\ 1/2 & 0 & 1/2 \\ 1/2 & 1/2 & 0 \end{pmatrix}$. Prove that the chain is irreducible and all the states are aperiodic and non null persistent. (8)

14. (a) (i) In a car washing centre, cars arrive at a rate of 30 per day. Washing time is exponentially distributed with an average of 40 minutes. Find

(1) The average queue length

(2) Probability that queue size exceeds 2 cars

(3) Probability that the centre has at least 4 cars

(4) The expected waiting time of a car in the centre. (8)

(ii) Derive L_s, L_q for the queuing model (M/M/1): (K/FIFO). (8)

Or

(b) There are three typists in an office. Each typist can type an average of 6 letters per hour.

If letters arrive for being typed at the rate of 15 letters per hour,

(i) What fraction of the time all the typists will be busy?

(ii) What is the average number of letters waiting to be typed?

(iii) What is the average time a letter has to spend for waiting and for being typed?

(iv) What is the probability that a letter will take longer than 20 minutes waiting to be typed and being typed? (16)

15. (a) Derive the formula for L_s, L_q, W_s, W_q for (M/G/1) queuing system. Deduce the formula for (M/M/1) queuing system. (16)

Or

(b) In a network of 3 service stations 1,2,3 customers arrive at 1,2,3, from outside, in accordance with Poisson process having rates 5, 10, 15 respecti ely. The service times at the 3 stations are exponential with respective rates 10, 50, 100. A customer completing service at station 1 is equally likely to

(i) Go to station 2,

(ii) Go to station 3 or

(iii) Leave the system. A customer departing from service at station 2 always goes to station 3. A departure from service at station 3 is equally likely to go to station 2 or leave the system. What is the average number of customers in the system, consisting of all the stations? And what is the average time a customer spends in the system? (16)