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Question Paper Code : 80611

B.E./B.Tech. DEGREE EXAMINATION, NOVEMBER/DECEMBER 2016.

Fourth Semester

Computer Science and Engineering

MA 6453 — PROBABILITY AND QUEUEING THEORY

(Common to Mechanical Engineering (Sandwich) and Information Technology)

(Regulations 2013)

Time : Three hours　　　　Maximum : 100 marks

Answer ALL questions.

PART A — ($10 \times 2 = 20$ marks)

1. If a fair coin is tossed twice, find $P(X \leq 1)$, where X denotes the number of heads in each experiment.

2. Balls are tossed at random into 50 boxes. Find the expected number of tosses required to get the first ball in the fourth box.

3. The joint probability density function of a random variable (X, Y) is $f(x, y) = ke^{-(2x+3y)}$, $x \geq 0$, $y \geq 0$. Find the value of k.

4. Write any two properties of joint cumulative distribution function.

5. A random process has the autocorrelation function $R_{xx}(\tau) = \frac{4\tau^2+6}{\tau^2+1}$, find the mean square value of the process.

6. Define Markov process.

7. What do you mean by balking, reneging of a queuing system?

8. State Little's formula for the queueing model $(M/M/1):(K/FIFO)$.

9. Write down the Pollaczek – Khintchine formula for $(M/G/1)$ queuing system.

10. Define bottle-neck of the system in queue networks.

PART B — (5 × 16 = 80 marks)

11. (a) (i) The probability distribution function of a random variable X is given by $f(x)=\frac{4x(9-x^2)}{81}$, $0 \le x \le 3$. Find the mean, variance and third moment about origin. (10)

(ii) Messages arrive at a switch board in a Poisson manner at an average rate of six per hour. Find the probability that exactly two messages arrive within one hour, no message arrives within one hour and at least three messages arrive within one hour. (6)

Or

(b) (i) State and prove forgetfulness property of exponential distribution. Using this property, solve the following problem :

The length of the shower on a tropical island during rainy season has an exponential distribution with parameter 2, time being measured in minutes. What is the probability that a shower will last more than 3 minutes? (10)

(ii) The annual rainfall in inches in a certain region has a normal distribution with a mean of 40 and variance of 16. What is the probability that the rainfall in a given year is between 30 and 48 inches? (6)

12. (a) (i) The joint CDF of two discrete random variables X and Y is given by

$$F(x, y)=\begin{cases}\frac{1}{8}, & x=1, y=1\\ \frac{5}{8}, & x=1, y=2\\ \frac{1}{4}, & x=2, y=1\\ 1, & x=2, y=2\end{cases}$$

. Find the joint probability mass function and the marginal probability mass functions of X and Y. (8)

(ii) If X and Y each follow an exponential distribution with parameter 1 and are independent, find the probability density function of $U=X-Y$. (8)

Or

(b) (i) The joint probability density function of a two dimensional random variable (X, Y) is given by $f(x, y)=\frac{1}{8}x(x-y)$, $0<x<2$, $-x<y<x$ and 0 elsewhere. Find the marginal distributions of X and Y and the conditional distribution of $Y=y$ given that $X=x$. (8)

(ii) Calculate the correlation coefficient for the following data : (8)

x:	65	66	67	67	68	69	70	72
y:	67	68	65	68	72	72	69	71

13. (a) (i) A random process $\{X(t)\} = K \cos \omega t$, $t \geq 0$ where K is uniformly distributed in $(0, 2)$. Determine the mean, autocorrelation and auto covariance function of the process $\{X(t)\}$. (10)

(ii) Prove that the inter arrival time of a Poisson process is an exponential distribution. (6)

Or

(b) (i) There are 2 white marbles in urn A and 3 red marbles in urn B. At each step of the process, a marble is selected from each urn and the 2 marbles selected are interchanged. Let the state a_i of the system be the number of red marbles in A after i changes. What is the probability that there are 2 red marbles in A after 3 steps? In the long run, what is the probability that there are 2 red marbles in urn A? (10)

(ii) Find the nature of the states of the Markov with the tpm $P = \begin{bmatrix} 0 & 1 & 0 \\ 0.5 & 0 & 0.5 \\ 0 & 1 & 0 \end{bmatrix}$. (6)

14. (a) There are three typists in an office. Each typist can type an average of 6 letters per hour. If letters arrive for being typed at the rate of 15 letters per hour :

(i) What fraction of time all the typist will busy?

(ii) What is the average number of letters waiting to be typed?

(iii) What is the average time a letter has to spend for waiting and for being typed?

(iv) What is the probability that a letter will take longer than 20 minutes waiting to be typed and being typed? (16)

Or

(b) Patients arrive at a clinic according to Poisson distribution at a rate of 30 patients per hour. The waiting room does not accommodate more than 14 patients. Examination time per patient is exponential with mean rate of 20 per hour :

(i) Find the effective arrival rate at the clinic.

(ii) What is the probability that an arriving patient will not wait?

(iii) What is the expected waiting time until a patient is discharged from the clinic? (16)

15. (a) Consider a queuing system where arrivals are according to a Poisson distribution with mean 5 per hour. Find the expected waiting time in the system if the service time distribution is a uniform distribution between $t = 5$ minutes and $t = 15$ minutes. (16)

Or

(b) Consider a system of two servers where customers from outside the system arrive at server 1 at a Poisson rate 4 and at server 2 at a Poisson rate 5. The service rates in 1 and 2 are respectively 8 and 10. A customer upon completion of service at server 1 is equally likely to go to server 2 or to leave the system; whereas a departure from server 2 will go 25% of the time to server 1 and will depart the system otherwise. Determine the limiting probabilities, average number of customers and average waiting time of a customer in the system. (16)

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