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Question Paper Code : 57508

B.E./B.Tech. DEGREE EXAMINATION, MAY/JUNE 2016

Fourth Semester

Computer Science and Engineering

MA 6453 – PROBABILITY AND QUEUEING THEORY

(Common to Mechanical Engineering (Sandwich) and Information Technology)

(Regulations 2013)

Time : Three Hours **Maximum : 100 Marks**

Use of statistical tables may be permitted.

Answer ALL questions.

PART – A (10 × 2 = 20 Marks)

1. Let X be a discrete R.V. with probability mass function

$$P(X = x) = \begin{cases} \dfrac{x}{10}, & x = 1, 2, 3, 4, \\ 0, & \text{otherwise} \end{cases}$$

Compute $P(X < 3)$ and $E\left(\frac{1}{2}X\right)$.

2. If a R.V X has the moment generating function $M_x(t) = \frac{3}{3-t}$, compute $E(X^2)$.

3. The joint p.d.f. of R.V. (X,Y) is given as $f(x, y) = \begin{cases} \dfrac{1}{x}, & 0 < y < x \leq 1 \\ 0, & \text{otherwise} \end{cases}$

Find the marginal p.d.f. of Y.

4. Let X and Y be two independent R.Vs with Var(X) = 9 and Var(Y) = 3. Find Var(4X – 2Y + 6).

5. The random process X(t) is given by $X(t) = Y\cos(2\pi t)$, $t > 0$, where Y is a R.V. with E(Y) = 1. Is the process X(t) stationary ?

6. Derive the autocorrelation function for a Poisson process with rate λ.

7. For an M/M/C/N FCFS ($C < N$) queueing system, write the expressions for P_0 and P_N.

8. Define (i) balking and (ii) reneging of the customers in the queueing system.

9. An M/D/1 queue has an arrival rate of 10 customers per second and a service rate of 20 customers per second. Compute the mean number of customers in the system.

10. Write a expression for the traffic equation of the open Jackson queueing network.

PART – B (5 × 16 = 80 Marks)

11. (a) (i) The probability mass function of a discrete R.V X is given in the following table :

X	–2	–1	0	1	2	3
$P(X = x)$	0.1	k	0.2	2k	0.3	k

Find (1) the value of k (2) $P(X < 1)$ (3) $P(-1 < X \leq 2)$ (4) E(X). **(8)**

(ii) Let X be a continuous R.V with probability density function

$$f(x) = \begin{cases} xe^{-x} & , x > 0 \\ 0 & , \text{otherwise} \end{cases}$$

Find (1) the cumulative distribution function of X

(2) Moment Generating Function $M_x(t)$ of X

(3) $P(X < 2)$

(4) E(X). **(8)**

OR

(b) (i) Let $P(X = x) = \left(\frac{3}{4}\right)\left(\frac{1}{4}\right)^{x-1}$, $x = 1, 2, 3, ...$, be the probability mass function of a R.V X Compute **(8)**

(1) $P(X > 4)$

(2) $P(X > 4/X > 2)$

(3) $E(X)$

(4) $Var(X)$

(ii) Let X be a uniformly distributed R.V. over $[-5, 5]$. Determine **(8)**

(1) $P(X \leq 2)$

(2) $P(|X| > 2)$

(3) Cumulative distribution function of X

(4) Var (X)

12. (a) (i) Find the constant k such that

$$f(x, y) = \begin{cases} k(x+1)e^{-y} & , 0 < x < 1 , y > 0 \\ 0 & , \text{otherwise} \end{cases}$$

is a joint p.d.f. of the continuous R.V. (X, Y). Are X and Y independent R.Vs ? Explain. **(8)**

(ii) The joint p.d.f. of the continuous R.V. (X, Y) is given as

$$f(x, y) = \begin{cases} e^{-(x+y)} & , x > 0 , y > 0 \\ 0 & , \text{otherwise} \end{cases}$$

Find the p.d.f. of the R.V $U = \frac{X}{Y}$. **(8)**

OR

(b) (i) Let the joint p.d.f. of R.V. (X, Y) be given as **(8)**

$f(x, y) = \begin{cases} Cxy^2 & , 0 \leq x \leq y \leq 1 \\ 0 & , \text{otherwise} \end{cases}$ Determine (1) the value of C (2) the marginal p.d.fs of X and Y (3) the conditional p.d.f. $f(x/y)$ of X given $Y = y$

(ii) A joint probability mass function of the discrete R.Vs X and Y is given as

$$P(X = x, Y = y) = \begin{cases} \frac{x+y}{32} & , x = 1,2, y = 1, 2, 3, 4 \\ 0 & , \text{otherwise} \end{cases}$$

Compute the covariance of X and Y.

13. (a) (i) Consider a random process $Y(t) = X(t) \cos(w_0 t + \theta)$, where X (t) is wide-sense stationary process, θ is a uniformly distributed R.V. over $(-\pi, \pi)$ and w_0 is a constant. It is assumed that X(t) and θ are independent. Show that Y(t) is a wide-sense stationary. **(8)**

(ii) Consider a Markov chain $\{X_n ; n = 0, 1, 2, \ldots\}$ having state space $S = \{1, 2\}$ and one-step

$$\text{TPM } P = \begin{bmatrix} \frac{4}{10} & \frac{6}{10} \\ \frac{8}{10} & \frac{2}{10} \end{bmatrix}.$$ **(8)**

(1) Draw a transition diagram.

(2) Is the chain irreducible ?

(3) Is the state-1 ergodic ? Explain.

(4) Is the chain ergodic ? Explain.

OR

(b) (i) Let X(t) and Y(t) be two independent Poisson processes with parameters λ_1 and λ_2 respectively. Find

(1) $P(X(t) + Y(t)) = n, \; n = 0, 1, 2, 3, \ldots,$

(2) $P(X(t) - Y(t)) = n, \; n = 0, \pm 1, \pm 2, \ldots.$ **(8)**

(ii) Consider a Markov chain $\{X_n; n = 0, 1, 2, \ldots\}$ having state space $S = \{1, 2, 3\}$ and one-step TPM $P = \begin{bmatrix} 0 & 1 & 0 \\ \frac{1}{2} & 0 & \frac{1}{2} \\ 1 & 0 & 0 \end{bmatrix}$ and initial probability distribution $P(X_0 = i) = \frac{1}{3}, i = 1, 2, 3.$

Compute

(1) $P(X_3 = 2, X_2 = 1, X_1 = 2 / X_0 = 1)$

(2) $P(X_3 = 2, X_2 = 1/X_1 = 2, X_0 = 1)$

(3) $P(X_2 = 2/X_0 = 2)$

(4) Invariant probabilities of the Markov chain. (8)

14. (a) (i) Customers arrive at a watch repair shop according to a Poisson process at a rate of 1 per every 10 minutes, and the service time is an exponential random variable with mean 8 minutes. Compute (8)

(1) the mean number of customers L_s in the system.

(2) the mean waiting time W_s of a customer spends in the system,

(3) the mean waiting W_q of a customer spends in the queue,

(4) the probability that the server is idle.

(ii) A petrol pump station has 4 petrol pumps. The service time follows an exponential distribution with mean of 6 minutes and cars arrive for service in a Poisson process at the rate of 30 cars per hour.

(1) Find the probability that no car is in the system.

(2) What is the probability that an arrival will have to wait in the queue ?

(3) Find the mean waiting time in the system. (8)

OR

(b) (i) A one person barber shop has 6 chairs to accommodate people waiting for a haircut. Assume that customers who arrive when all the 6 chairs are full leave without entering the barber shop. Customers arrive at the rate of 3 per hour and spend an average of 15 minutes in the barber's chair. Compute

(1) P_0

(2) L_q

(3) P_7

(4) W_s **(8)**

(ii) Consider a single-server queue where the arrivals are Poisson with rate $\lambda = 10$ / hour. The service distribution is exponential with rate $\mu = 5$/hour. Suppose that customers balk at joining the queue when it is too long. Specifically, when there are 'n' in the system, an arriving customer joins the queue with probability $\frac{1}{(n+1)}$. Determine the steady-state probability that there are 'n' customers in the system. **(8)**

15. (a) Discuss an M/G/1/∞ FCFS queueing system and hence obtain the Pollaczek-Khintchine (P-K) mean value formula. Deduce also the mean system size for the M/M/1/∞ : FCFS queueing system from the P-K formula. **(16)**

OR

(b) (i) The police department has 5 patrol cars. A patrol car breaks down and repairs service one every 30 days. The police department has two repair workers, each of whom takes an average of 3 says to repair a car. Breakdown times and repair time are exponential. Determine the average number of patrol cars in good condition. Also find the average down time for a patrol car that needs repairs. **(8)**

(ii) A repair facility is shared by a large by a large number of machines for repair. The facility has two sequential stations with respective rates of service 1 per hour and 3 per hour. The cumulative failure rate of all the machines is 0.5 per hour. Assuming that the system behaviour may be approximated by a two-station tandem queue. Find

(1) the average number of customers in both station,

(2) the average repair time,

(3) the probability that both service stations are idle. **(8)**
