ANNA UNIVERSITY COIMBATORE

B.E. / B.TECH. DEGREE EXAMINATIONS : DECEMBER 2009

REGULATIONS : 2007

FIFTH SEMESTER

070030022 - PROBABILITY AND QUEUEING THEORY

(COMMON TO CSE / IT)

TIME : 3 Hours Max.Marks : 100

PART – A

(20 x 2 = 40 MARKS)

ANSWER ALL QUESTIONS

1. Prove that the probability of the impossible event is zero.
2. Define Total Probability.
3. A continuous Random variable X that can assume any value between x = 2 and x = 5 has a density function given by f (x) = k (1 + x) . Find P(x < 4).
4. Define Random Variable. Give an example.
5. The moment generating function of a random variable X is given by $M_X(t) = e^{3(e^t-1)}$. Find P (X = 1).
6. Drive the recurrence formula for Geometric Distribution.
7. Find the mean of Uniform Density Function.
8. In a Normal Distribution whose mean is 12 and standard deviation is 2. Find the probability interval from x = 9.6 to x = 13.8.
9. The two equations of the variable X and Y are x=19.13 - 0.87y and y =11.64 - 0.50x. Find the correlation co-efficient between X and Y.

10. If X and Y are independent random variables with variance 2 and 3, find the variance of 3 X + 4 Y.
11. The joint p.d.f of a bivariate random variable (X,Y) is given by $f(x,y)=\begin{cases} kxy, & 0<x<1, 0<y<1 \\ 0, & otherwise \end{cases}$. Find the value of k.
12. Define Marginal Probability Density function.
13. Define Wide-sense Stationary Process.
14. State any two properties of Poisson Process
15. If the transition probability matrix of a Markov chain is $\begin{bmatrix} 0 & 1 \\ 1/2 & 1/2 \end{bmatrix}$, find the limiting distribution of the chain.
16. State Chapman-Kolmogorow theorem.
17. For $(M/M/1):(\infty/FIFO)$ model, write down the Little's formulae.
18. In a given $(M/M/1):(\infty/FCFS)$ queue, p=0.6. What is the probability that the queue contains 5 or more customers?
19. Explain your understanding of the relationship between the arrival rate λ and average arrival time.
20. In (M/G/1) model ,what is the formula for the average number of customers in the system?

PART – B

(5 x 12 = 60 MARKS)

ANSWER ANY FIVE QUESTIONS

21. a The contents of urns I, II, III are as follows: 2 white, 3 black and 4 red balls; 3 white,2 black and 2 red balls and 4 white, 1 black and 3 red balls. An urn is chosen at random and two balls are drawn. They happen to be white and red. What is the probability that they come from urns I, II or III? 6

b A random variable X has the following probability function 6

X:	0	1	2	3	4
P(X):	K	3K	5K	7K	9K

(i) Find the value K

(ii) Find P(X < 3) and P(0 < X < 4)

(iii) Find the distribution function of X.

22. a Find the mean, variance and moment generating function of a binomial distribution. 6

b The p.d.f. of the length of the time that a person speaks over phone is 6

$$f_X(x)\begin{cases} Be^{\frac{-x}{6}}, & x > 0 \\ 0, & otherwise \end{cases}$$

. What is the probability that the person will talk for (i) more than 8 minutes (ii) less than 4 minutes (iii) between 4 & 8 minutes.

23. a From the following data, find the regression equation of x on y. 6

x:	25	28	35	32	31	36	29	38	34	32
y:	43	46	49	41	36	32	31	30	33	39

b If X andY are two random variables having joint density function 6

$$f(x,y)=\begin{cases}\frac{1}{8}(6-x-y), & 0<x<2,\ 2<y<4 \\ 0\ , & otherwise\end{cases},$$

find $P(X<1 \cap Y<3)$ and $P(X<1\ /\ Y<3)$.

24. a The one step transition probability matrix of a Markov Chain $\{X_n\,;\, n=0,1,2,...\}$ having state space S = $\{1,2,3\}$ is P = $\begin{bmatrix}0.1 & 0.5 & 0.4\\ 0.6 & 0.2 & 0.2\\ 0.3 & 0.4 & 0.3\end{bmatrix}$ and the initial distribution is $\pi_0 = (0.7, 0.2, 0.1)$. Find (i) $P(X_2=3/X_0=1)$ 6

(ii) $P(X_2=3)$

(iii) $P(X_3=2, X_2=3, X_1=3, X_0=1)$.

b Show that the random process $X(t)= A\cos(\omega t+\theta)$ is wide sense stationary if A and ω are constant and θ is uniformly distributed random variable in $(0, 2\pi)$. 6

25. a The capacity of a communication line is 2,000 bits per second. This line is used to transmit eight-bit characters, so the maximum rate is 250 characters per second. The application calls for traffic from many devices to be sent on the line with a total volume of 12,000 characters per minute. Determine 6

Cont....Q.No.25.(a)

(i) the line utilization

(ii) the average number of characters waiting to be transmitted

(iii) the average transmission(including queueing delay) time per characters.

25. b A supermarket has two girls serving at the counters. The customers arrive in Poisson fashion at the rate of 12 per hour. The service time for each customer is exponential with mean 6 minutes. 6

Find (i) the probability that an arriving customer has to wait for service

(ii) the average number of customers in the system

(iii) the average time spent by a customers in the supermarket.

26. a A continuous random variable X has the p.d.f $f(x)=K\,x^2 e^{-x}\ ,x\geq 0$. Find the r^{th} moment of X about the origin. Hence find the mean and variance of X. 6

b If the random variable X with density function $f(x)=\begin{cases}2x\ ,0<x<1\\ 0\ \ ,\ otherwise\end{cases}$, find the p.d.f of $Y=8X^3$. 6

27. Obtain the expression for steady state probabilities of M/M/C queueing system

28. a If the joint p.d.f of (X,Y) is given by $f_{XY}\ (x,y)=x+y,\ 0\leq x,\ y\leq 1$, find the p.d.f of U = XY. 6

b Six coins are tossed 6400 times. Using the Poisson distribution, what is the approximate probability of getting six heads 10 times. 6