Reg. No. : | | | | | | | | | | | |

Question Paper Code : 51574

B.E./B.Tech. DEGREE EXAMINATION, MAY/JUNE 2014.

Fourth Semester

Computer Science and Engineering

MA 2262/MA 44/MA 1252/080250008/10177 PQ 401 – PROBABILITY AND QUEUEING THEORY

(Common to Information Technology).

(Regulation 2008/2010)

Time : Three hours   Maximum : 100 marks

(Use of statistical tables may be permitted)

Answer ALL questions.

PART A — (10 × 2 = 20 marks)

1. A continuous random variable X has the probability density function given by

$$f(x) = \begin{cases} a(1 + x^2), & 2 \le x \le 5 \\ 0 & \text{otherwise} \end{cases}$$

Find a and $P(X < 4)$.

2. For a binomial distribution with mean 6 and standard deviation $\sqrt{2}$, find the first two terms of the distribution.

3. Find the value of k, if the joint density function of (X, Y) is given by $f(x, y) = \begin{cases} k(1 - x)(1 - y), & 0 < x < 4, 1 < y < 5 \\ 0, & \text{otherwise} \end{cases}$

4. Given that joint probability density function of (X, Y) as $f(x, y) = \frac{1}{6}, 0 < x < 2, 0 < y < 3$, determine the marginal density.

5. Define strict sense and wide sense stationary process.

6. A gambler has Rs. 2. He bets Re. 1 at a time and wins Re. 1 with probability 1/2. He stops playing if he loses Rs. 2 or wins Rs. 4. What is the transition probability matrix of the related Markov chain?

7. A supermarket has a single cashier. During peak hours, customers arrive at a rate of 20 per hour. The average number of customers that can be serviced by the cashier is 24 per hour. Calculate the probability that the cashier is idle.

8. State the steady state probabilities of the finite source queueiug model represented by $(M/M/R):(GD/K/K)$.

9. State Pollaczek-Khintchine formula for the average number in the system in a $M/G/1$ queueing model and hence derive the same when the service time is constant with mean $\frac{1}{\mu}$.

10. Distinguish between open and closed queueing network.

PART B — $(5 \times 16 = 80$ marks)

11. (a) (i) Suppose that a trainee soldier shoots a target in an independent fashion. If the probability that the target is shot on any one shot is 0.7,

(1) What is the probability that the target would be hit on tenth attempt?

(2) What is the probability that it takes him less than 4 shots?

(3) What is the probability that it takes him an even number of shots? (8)

(ii) Determine the moment generating function of an exponential random variable and hence find its mean and variance. (8)

Or

(b) (i) Determine the mean, variance and moment generating function of a random variable X following Poisson distribution with parameter λ. (8)

(ii) Trains arrive at a station at 15 minutes intervals starting at 4 a.m. If a passenger arrive at a station at a time that is uniformly distributed between 9.00 and 9.30, find the probability that he has to wait for the train for (1) less than 6 minutes (2) more than 10 minutes. (8)

12. (a) (i) The joint probability density function of two random variables X and Y is given by $f(x,y) = \begin{cases} \frac{6}{7}\left(x^2 + \frac{xy}{2}\right), & 0 \le x \le 1, 0 \le y \le 2 \\ 0, & \text{otherwise} \end{cases}$

Find the conditional density function of X given Y and the conditional density function of Y given X. (8)

(ii) If the independent random variables X and Y have the variances 36 and 16 respectively, find the correlation coefficient, r_{UV} where $U = X + Y$ and $V = X - Y$. (8)

Or

(b) The joint probability density function of two random variables X and Y is given by $f(x, y) = \begin{cases} k[(x + y) - (x^2 + y^2)], & 0 < (x, y) < 1 \\ 0 & \text{otherwise} \end{cases}$

Show that X and Y are uncorrelated but not independent. (16)

13. (a) (i) Let $\{X_n\}$ be a Markov chain with state space {0, 1, 2} with initial probability vector $p^{(0)} = (0.7, 0.2, 0.1)$ and the one step transition probability matrix $P = \begin{pmatrix} 0.1 & 0.5 & 0.4 \\ 0.6 & 0.2 & 0.2 \\ 0.3 & 0.4 & 0.3 \end{pmatrix}$. Compute $P(X_2 = 3)$ and $P(X_3 = 2, X_2 = 3, X_1 = 3, X_0 = 2)$. (8)

(ii) If $\{X_1(t)\}$ and $\{X_2(t)\}$ are two independent Poisson processes with parameters λ_1 and λ_2 respectively, show that the process $\{X_1(t) + X_2(t)\}$ is also a Poisson process. (8)

Or

(b) (i) Show that the random process $X(t) = A\cos(\omega t + \theta)$ is a Wide Sense Stationary process if A and ω are constants and θ is a uniformly distributed random variable in $(0, 2\pi)$. (8)

(ii) Consider a Markov chain with transition probability matrix $P = \begin{pmatrix} 0.5 & 0.4 & 0.1 \\ 0.3 & 0.4 & 0.3 \\ 0.2 & 0.3 & 0.5 \end{pmatrix}$. Find the limiting probabilities of the system. (8)

14. (a) (i) Derive the governing equations for the $(M / M / 1) : (GD / N / \infty)$ queueing model and hence obtain the expression for the steady state probabilities and the average number of customers in the system. (10)

(ii) Four counters are being run on the frontiers of the country to check the passports of the tourists. The tourists choose a counter at random. If the arrival at the frontier is Poisson at the rate λ and the service time is exponential with parameter $\lambda/2$, find the average queue length at each counter. (6)

Or

(b) (i) Derive the governing equation for the $(M/M/C):(GD/\infty/\infty)$ queueing model and hence obtain the expression for the steady state probabilities and the average number of customers in the queue. (10)

(ii) Customers arrive at the express checkout lane in a supermarket in a Poisson process with a rate of 15 per hour. The time to check-out a customer is an exponential random variable with mean of 2 minutes. Find the average number of customers present. What is the expected waiting time for a customer in the system? (6)

15. (a) (i) An automatic car wash facility operates with only one bay. Cars arrive according to a Poisson distribution with a mean of 4 cars/hour and may wait in the facility's parking lot if the bay is busy. Find the average number of customers in the system in the service time is (1) constant and is equal to 10 minutes (2) uniformly distributed between 8 and 12 minutes. (8)

(ii) An average of 120 students arrive each hour (inter-arrival times are exponential) at the controller office to get their hall tickets. To complete this process, a candidate must pass through three counters. Each counter consists of a single server. Service times at each counter are exponential with the following mean times: counter 1,20 seconds; counter 2, 15 seconds and counter 3, 12 seconds. On the average, how many students will be present in the controller's office. (8)

Or

(b) Consider a open queueing network with parameter values shown below :

Facility j	s_j	μ_j	a_j	$i = 1$	$i = 2$	$i = 3$
$j = 1$	1	10	1	0	0.1	0.4
$j = 2$	2	10	4	0.6	0	0.4
$j = 3$	1	10	3	0.3	0.3	0

(i) Find the steady state distribution of the number of customers at facility 1, facility 2 and facility 3.

(ii) Find the expected total number of customers in the system.

(iii) Find the expected total waiting time for a customer. (16)