Reg. No. | | | | | | | | | | | | |

Question Paper Code : 51774

B.E./B. Tech. DEGREE EXAMINATION, MAY/JUNE 2016

Fourth Semester

Computer Science and Engineering

MA2262/MA1252/MA44/10177PQ401/080250008 – PROBABILITY AND QUEUEING THEORY

(Common to Information Technology)

(Regulation 2008/2010)

Time : Three Hours

Maximum : 100 Marks

Answer ALL questions.

PART – A (10 × 2 = 20 Marks)

1. Suppose that a continuous random variable X has the probability density function $f(x) = \begin{cases} K(1-x^2) & \text{for } 0 < x < 1 \\ 0 & \text{elsewhere} \end{cases}$. Find the value of K.

2. If x is a binomially distributed random variable with $E(x) = 2$ and $Var(x) = \frac{4}{3}$, then find the probability mass function of x.

3. Define independence of two discrete random variables X and Y.

4. Find the probability distribution of X + Y from the bivariate distribution of (X, Y) given below :

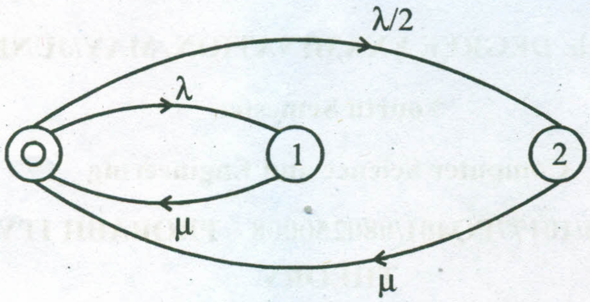
X \ Y	1	2
1	0.4	0.2
2	0.3	0.4

5. What do you mean by a homogeneous Markov chain ?

6. What do you mean by an absorbing state ?

7. What is meant by a transient state in Queueing Theory ?

8. Write the balance equation for the state O from the following diagram in steady state :



9. What do you mean by non-Markovian Queueing Model ?

10. What is meant by closed queueing network ?

PART – B ($5 \times 16 = 80$ Marks)

11. (a) (i) For the following density function $f(x) = ae^{-|x|}$; $-\infty < x < \infty$, find the value of a, mean and variance. **(8)**

(ii) Find the moment generating function of a geometric random variable. Also find its mean. **(8)**

OR

(b) (i) Assume that 50% of all engineering students are good in mathematics. Determine the probabilities that among 18 engineering students (1) exactly 10 (2) at least 10 are good in Maths. **(8)**

(ii) The life (in years) of a certain electrical switch has an exponential distribution with an average life of $\frac{1}{\lambda} = 2$. If 100 of these switches are installed in different systems, find the probability that at most 30 fail during the first year. **(8)**

12. (a) (i) Find the coefficient of correlation between X and Y using the following data : **(8)**

X :	5	10	15	20	25
Y :	16	19	23	26	30

(ii) If X and Y each follow an exponential distribution with parameter 1 and are independent, find the pdf of $U = X - Y$. **(8)**

OR

(b) The joint pdf of random variables X and Y is given by

$$f(x, y) = \begin{cases} \lambda x y^2; & o \le x \le y \le 1 \\ 0; & \text{otherwise} \end{cases}$$

(i) Find the value of λ.

(ii) Find the marginal pdfs of X and Y.

(iii) Calculate E(X) and E(Y).

(iv) Are X and Y independent ? Justify. **(16)**

13. (a) (i) Suppose that earthquakes occur in a certain region of California, in accordance with a Poisson process, at a rate of seven per year. What is the probability of no earthquakes in one year ? What is the probability that is exactly 3 of the next eight years no earthquakes will occur ? **(8)**

(ii) A fair die is tossed repeatedly. The maximum of the first n outcomes is denoted by X_n. Calculate its t.p.m. and p^2. **(8)**

OR

(b) (i) An observer at a lake notices that when fish are caught, only 1 out of 9 trout is caught after another trout, with no other fish between, whereas 10 out of 11 non-trout are caught following non-trout, with no trout between. Assuming that all fish are equally likely to be caught, what fraction of fish in the lake is trout ? **(8)**

(ii) At an intersection, a working traffic light will be out of order the next day with probability 0.07, and an out of order traffic light will be working the next day with probability 0.88. Identify the state space and t.p.m. Also find $P(X_2 = 1)$. **(8)**

14. (a) A telephone company is planning to install telephone booths in a new airport. It has established the policy that a person should not have to wait more than 10% of the times he tries to use a phone. The demand for use is estimated to be Poisson with an average of 30 per hour. The average phone call has an exponential distribution with a mean of 5 mins. How many phone booths should be installed ? **(16)**

OR

(b) If people arrive to purchase cinema tickets at the average rate of 6 per minute, it takes an average of 7.5 seconds to purchase a ticket. If a person arrives 2 min. before the picture starts and if it takes exactly 1.5 min to reach the correct seat after purchasing the ticket, (i) can be expect to be seated for the start of the picture ? (ii) What is the probability that he will be seated for the start of the picture ? (iii) How early must he arrive in order to be 99% sure of being seated for the start of the picture ? **(16)**

15. (a) Derive Pollaczek-Khintchine formula of an M/G/1 queueing model. **(16)**

OR

(b) Write a short note on :

(i) Open queueing network **(10)**

(ii) Closed queueing network **(6)**
