Reg. No. : | | | | | | | | | | | |

Question Paper Code : 10397

B.E./B.Tech. DEGREE EXAMINATION, MAY/JUNE 2012.

Fourth Semester

Computer Science and Engineering

MA 2262/181404/MA 44/MA 1252/10177 PQ 401/080250008 — PROBABILITY AND QUEUING THEORY

(Common to Information Technology)

(Regulation 2008)

Time : Three hours | Maximum : 100 marks

Statistical Tables may be permitted.

Answer ALL questions.

PART A — (10 × 2 = 20 marks)

1. Check whether the following is a probability density function or not :

$$f(x)=\begin{cases}\lambda e^{-\lambda x} & x\geq 0,\ \lambda>0\\ 0 & \text{elsewhere.}\end{cases}$$

2. If a random variable has the moment generating function given by $M_X(t)=\dfrac{2}{2-t}$, determine the variance of X.

3. The regression equations of X on Y and Y on X are respectively $5x-y=22$ and $64x-45y=24$. Find the means of X and Y.

4. State Central limit theorem.

5. Define Wide sense stationary process.

6. If the initial state probability distribution of a Markov chain is $p^{(0)}=\begin{bmatrix}\frac{5}{6} & \frac{1}{6}\end{bmatrix}$ and the transition probability matrix of the chain is $\begin{bmatrix}0 & 1\\ 1/2 & 1/2\end{bmatrix}$, find the probability distribution of the chain after 2 steps.

7. State Little's formula for a $(M/M/1):(GD/N/\infty)$ queuing model.

8. Define steady state and transient state in Queuing theory.

9. When a M/G/1 queuing model will become a classic M/M/1 queuing model?

10. State Pollaczek-Khintchine formula for the average number of customers in a M/G/1 queuing model.

PART B — (5 × 16 = 80 marks)

11. (a) (i) A random variable X has the following probability function :

X:	0	1	2	3	4	5	6	7
$P(X)$:	0	k	$2k$	$2k$	$3k$	k^2	$2k^2$	$7k^2+k$

(1) Find the value of k.

(2) Evaluate $P(X<6)$, $P(X \geq 6)$.

(3) If $P(X \leq c) > \frac{1}{2}$ find the minimum value of c. (8)

(ii) Find the moment generating function of an exponential random variable and hence find its mean and variance. (8)

Or

(b) (i) If X is a Poisson variate such that $P(X=2) = 9P(X=4) + 90P(X=6)$. Find

(1) Mean and $E(X^2)$

(2) $P(X \geq 2)$. (8)

(ii) In a certain city, the daily consumption of electric power in millions of kilowatt hours can be treated as a random variable having Gamma distribution with parameters $\lambda = \frac{1}{2}$ and $v = 3$. If the power plant of this city has a daily capacity of 12 millions kilowatt-hours, what is the probability that this power supply will be inadequate on any given day? (8)

12. (a) (i) Let X and Y be two random variables having the joint probability function $f(x, y) = k(x+2y)$ where x and y can assume only the integer values 0, 1 and 2. Find the marginal and conditional distributions. (8)

(ii) Two random variables X and Y have the joint probability density function

$$f(x, y) = \begin{cases} c(4-x-y), & 0 \leq x \leq 2,\ 0 \leq y \leq 2 \\ 0, & \text{elsewhere}. \end{cases}$$

Find $\operatorname{cov}(X, Y)$. (8)

Or

(b) (i) Two dimensional random variable (X, Y) have the joint probability density function

$$f(x, y) = 8xy, \quad 0 < x < y < 1$$
$$= 0, \quad \text{elsewhere}.$$

(1) Find $P\left(X < \frac{1}{2} \cap Y < \frac{1}{4}\right)$.

(2) Find the marginal and conditional distributions.

(3) Are X and Y independent? (8)

(ii) Suppose that in a certain circuit, 20 resistors are connected in series. The mean and variance of each resistor are 5 and 0.20 respectively. Using Central limit theorem, find the probability that the total resistance of the circuit will exceed 98 ohms assuming independence. (8)

13. (a) (i) The process $\{X(t)\}$ whose probability distribution under certain condition is given by

$$P[X(t)=n]=\frac{(at)^{n-1}}{(1+at)^{n+1}} \quad n=1, 2, 3, \ldots$$

$$=\frac{at}{1+at} \quad n=0.$$

Show that $\{X(t)\}$ is not stationary. (8)

(ii) A salesman territory consists of three cities A, B and C. He never sells in the same city on successive days. If he sells in city-A, then the next day he sells in city-B. However if he sells in either city-B or city-C, the next day he is twice as likely to sell in city-A as in the other city. In the long run how often does he sell in each of the cities? (8)

Or

(b) (i) The transition probability matrix of a Markov chain $\{X_n\}$, $n=1, 2, 3, \ldots$ having three states 1, 2 and 3 is $P=\begin{bmatrix}0.1 & 0.5 & 0.4\\ 0.6 & 0.2 & 0.2\\ 0.3 & 0.4 & 0.3\end{bmatrix}$ and the initial distribution is $p^{(0)}=(0,7, 0.2, 0.1)$. Find

(1) $P[X_2=3]$

(2) $P[X_3=2, X_2=3, X_1=3, X_0=2]$. (8)

(ii) Suppose that customers arrive at a bank according to Poisson process with mean rate of 3 per minute. Find the probability that during a time interval of two minutes

(1) exactly 4 customers arrive

(2) greater than 4 customers arrive

(3) fewer than 4 customers arrive. (8)

14. (a) (i) A T.V. repairman finds that the time spent on his job has an exponential distribution with mean 30 minutes. If he repair sets in the order in which they came in and if the arrival of sets is approximately Poisson with an average rate of 10 per 8 hour day. Find

(1) the repairman's expected idle time each day

(2) how many jobs are ahead of average set just brought? (8)

(ii) A supermarket has 2 girls running up sales at the counters. If the service time for each customer is exponential with mean 4 minutes and if people arrive in Poisson fashion at the rate of 10 per hour, find the following :

(1) What is the probability of having to wait for service?

(2) What is the expected percentage of idle time for each girl?

(3) What is the expected length of customer's waiting time? (8)

Or

(b) (i) Trains arrive at the yard every 15 minutes and the service time is 33 minutes. If the line capacity of the yard is limited to 5 trains, find the probability that the yard is empty and the average number of trains in the system, given that the inter arrival time and service time are following exponential distribution. (8)

(ii) There are three typists in an office. Each typist can type an average of 6 letters per hour. If letters arrive for being typed at the rate of 15 letters per hour, what fraction of times all the typists will be busy? What is the average number of letters waiting to be typed? (8)

15. (a) (i) Automatic car wash facility operates with only one bay. Cars arrive according to a Poisson distribution with a mean of 4 cars per hour and may wait in the facility's parking lot if the bay is busy. The parking lot is large enough to accommodate any number of cars. If the service time for all cars is constant and equal to 10 minutes, determine

(1) mean number of customers in the system L_s

(2) mean number of customers in the queue L_q

(3) mean waiting time of a customer in the system W_s

(4) mean waiting time of a customer in the queue W_q. (8)

(ii) An average of 120 students arrive each hour (inter-arrival times are exponential) at the controller office to get their hall tickets. To complete the process, a candidate must pass through three counters. Each counter consists of a single server, service times at each counter are exponential with the following mean times : counter 1, 20 seconds; counter 2, 15 seconds and counter 3, 12 seconds. On the average how many students will be present in the controller's office. (8)

Or

(b) (i) Derive the P-K formula for the $(M/G/1):(GD/\infty/\infty)$ queueing model and hence deduce that with the constant service time the P-K formula reduces to $L_s = \rho + \dfrac{\rho^2}{2(1-\rho)}$ where $\mu = \dfrac{1}{E(T)}$ and $\rho = \dfrac{\lambda}{\mu}$. (10)

(ii) For a open queueing network with three nodes 1, 2 and 3, let customers arrive from outside the system to node j according to a Poisson input process with parameters r_j and let P_{ij} denote the proportion of customers departing from facility i to facility j. Given $(r_1, r_2, r_3) = (1, 4, 3)$ and

$$P_{ij} = \begin{pmatrix} 0 & 0.6 & 0.3 \\ 0.1 & 0 & 0.3 \\ 0.4 & 0.4 & 0 \end{pmatrix}$$

determine the average arrival rate λ_j to the node j for $j = 1, 2, 3$.