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Question Paper Code : 57504

B.E./B.Tech. DEGREE EXAMINATION, MAY/JUNE 2016

Fourth Semester

Electronics and Communication Engineering

MA 6451 – PROBABILITY AND RANDOM PROCESSES

(Common to Biomedical Engineering, Robotics and Automation Engineering)

(Regulations 2013)

Time : Three Hours **Maximum : 100 Marks**

Answer ALL questions.

PART – A (10 × 2 = 20 Marks)

1. If $f(x) = \begin{cases} K e^{-x}, & x > 0 \\ 0, & \text{otherwise} \end{cases}$

   is the pdf of a random variable X, then find the value of K.

2. Let X be a random variable with moment generating function $M_X(t) = \frac{(2e^t + 1)^4}{81}$. Then find its mean and variance.

3. Let (X, Y) be a two-dimensional random variable. Define covariance of (X, Y). If X and Y are independent, what will be the covariance of (X, Y) ?

4. If the joint pdf of (X, Y) is

   $$f(x, y) = \begin{cases} \frac{1}{4}, & 0 < x, y < 2 \\ 0, & \text{otherwise} \end{cases}$$

   find $P(X + Y \leq 1)$

5. Define a stationary process.

6. What is Markov Process ?

7. The Power Spectral density of a random process $x(t)$ is given by

$$S_{xx}(w) = \begin{cases} \pi & \text{if } |w| < 1 \\ 0 & \text{elsewhere} \end{cases}$$

Find its autocorrelation function :

8. Prove that $R_{xy}(\tau) = R_{yx}(-\tau)$

9. Find the autocorrelation function of the white noise.

10. If the system has the impulse response

$$h(t) = \begin{cases} \dfrac{1}{2C} & , |+| \leq C \\ 0 & , |+| > C \end{cases}$$

Write down the relation between the spectrums of input X(t) and output Y(t).

PART – B (5 × 16 = 80 Marks)

11. (a) (i) State and prove memoryless property for geometric distribution. **(8)**

(ii) In a certain city, the daily consumption of electric power in millions of Kilowatt-hours can be considered as a random variable following gamma distribution with parameters $\lambda = \frac{1}{2}$ and $\alpha = 3$. If the power plant in this city has a daily capacity of 12 million Kilowatt-hours, what is the probability that this supply of power will be insufficient on any given day ? **(8)**

OR

(b) (i) A coin is biased so that a head is twice as likely to appear as a tall. If the coin is tossed 6 times, find the probabilities of getting (1) exactly 2 heads, (2) at least 3 heads, (3) at most 4 heads. **(8)**

(ii) The length of time a person speaks over phone follows exponential distribution with mean 6 mins. What is the probability that the person will talk for (1) more than 8 mins, (2) between 4 and 8 mins. **(8)**

12. (a) (i) Let (x, y) be a two-dimensional non-negative continuous random variable having the joint density.

$$f(x, y) = \begin{cases} 4xy\, e^{-(x^2 + y^2)} & , x \geq 0, y \geq 0 \\ 0 & , \text{otherwise} \end{cases}$$

Find the density function of $U = \sqrt{x^2 + y^2}$. **(8)**

(ii) Given : **(8)**

$$f_{xy}(x, y) = \begin{cases} Cx(x - y) & , 0 < x < 2, -x < y < x \\ 0 & , \text{otherwise} \end{cases}$$

(1) Evaluate C

(2) Find $f_x(x)$

(3) $f_{y/x}(y/x)$ and

(4) $f_y(y)$.

OR

(b) (i) Three balls are drawn at random without replacement from a box containing 2 white, 3 red and 4 black balls. If x denotes the number of white balls drawn and y denotes the number of red balls drawn, find the joint probability distribution of (x, y). (8)

(ii) In a partially destroyed laboratory record only the lines of regressions and variance of x are available. The regression equations are $8x - 10y + 66 = 0$ and $40x - 18y = 214$ and variance of $x = 9$. Find (1) the correlation coefficient between x and y (2) Mean values of x and y (3) Variance of y. (8)

13. (a) (i) Show that the random process $X(t) = A\cos(wt + \theta)$ is wide-sense stationary, where A and w are constants and θ is uniformly distributed on the interval $(0, 2\pi)$. (8)

(ii) A man either drives a car or catches a train to go to office each day. He never goes 2 days in a row by train but if he drives one day, then the next day he is just as likely to drive again as he is to travel by train. Now suppose that on the first day of the week, the mass tossed a fair die and drove to work if and only if 6 appeared. Find (1) the probability that he takes a train on the third day and (2) the probability that he drives to work in the long run. (8)

OR

(b) (i) Prove that the difference of two independent Poisson processes is not a Poisson process. (8)

(ii) Prove that a random telegraph signal process $y(t) = \alpha\, x(t)$ is a WSS process, where α is a random variable which is independent of $x(t)$, assumes values -1 and 1 with equal probability and $R_{xx}(t_1, t_2) = e^{-2\lambda |t_1 - t_2|}$. (8)

14. (a) (i) The autocorrelation function of an ergodic process $x(t)$ is $R_{xx}(\tau) = \begin{cases} 1 - |\tau|, & \text{if } |\tau| \le 1 \\ 0, & \text{otherwise} \end{cases}$

Obtain the spectral density of $x(t)$. (8)

(ii) The cross power spectrum of real random processes $x(t)$ and $y(t)$ is given by $S_{xy}(w) = \begin{cases} a + ibw, & \text{if } |w| < 1 \\ 0, & \text{elsewhere} \end{cases}$

Find the cross correlation function. (8)

OR

(b) (i) If $\{x(t)\}$ and $\{y(t)\}$ are two random processes with autocorrelation functions $R_{xx}(\tau)$ and $R_{yy}(\tau)$ respectively and jointly WSS, then prove that $|R_{xy}(\tau)| \leq \sqrt{R_{xx}(0) \cdot R_{yy}(0)}$. Establish any two properties of autocorrelation function $R_{xx}(\tau)$. (8)

(ii) Given the power spectral density of a continuous process as $S_{xx}(w) = \dfrac{w^2 + 9}{w^4 + 5w^2 + 4}$. Find the mean square value of the process. (8)

15. (a) (i) If $Y(t) = A \cos(w_o t + \theta) + N(t)$, where A is a constant, θ is a random variable with uniform distribution is $(-\pi, \pi)$ and N(t) is a band-limited Gaussian white noise with a power spectral density

$$S_{NN}(w) = \begin{cases} \dfrac{N_o}{2} & \text{for } |w - w_o| < W_B \\ 0 & \text{, elsewhere} \end{cases}$$

Find the power spectral density of Y(t).

Assume that N(t) and θ are independent. (10)

(ii) A linear time invariant system has an impulse response $\hbar(t) = e^{-Bt}\,U(t)$. Find the power spectral density of the output Y(t) corresponding to the input X(t). (6)

OR

(b) (i) Assume a random process X(t) is given as input to a system with transfer function $H(w) = 1$ for $-w_0 < w < w_0$. If the autocorrelation function of the input process is $\dfrac{N_o}{2}\,\delta(T)$, find the autocorrelation function of the output process. (8)

(ii) A circuit has unit impulse response given by

$$\hbar(t) = \begin{cases} \dfrac{1}{T} & ; \ 0 \leq t \leq T \\ 0 & ; \ \text{otherwise} \end{cases}$$

Evaluate $S_{yy}(w)$ interms of $S_{xx}(w)$. (8)
