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B.E./B.Tech. DEGREE EXAMINATION, NOVEMBER/DECEMBER 2015.

Fourth Semester

Electronics and Communication Engineering

MA 6451 — PROBABILITY AND RANDOM PROCESSES

(Common to Biomedical Engineering, Robotics and Automation Engineering)

(Regulations 2013)

Time : Three hours Maximum : 100 marks

Answer ALL questions.

PART A — (10 × 2 = 20 marks)

1. Assume that X is a continuous random variable with the probability density function $f(x) = \begin{cases} \frac{3}{4}(2x - x^2), 0 < x < 2 \\ 0 \quad \text{otherwise} \end{cases}$. Find $P(X > 1)$.

2. A random variable X is uniformly distributed between 3 and 15. Find the variance of X.

3. The joint probability mass function of a two dimensional random variable (X, Y) is given by $P(x, y) = K(2x + y)$, $x = 1, 2$ and $y = 1, 2$ where K is a constant. Find the value of K.

4. The two regression equations of two random variables X and Y are $4x - 5y + 33 = 0$ and $20x - 9y = 107$. Find the mean values of X and Y.

5. Define SSS process.

6. Write down any two properties of Poisson process.

7. Compute the mean value of the random process $\{X(t)\}$ whose auto correlation function is given by $\{R_{XX}(t)\} = 25 + \frac{4}{1 + 6\tau^2}$.

8. Prove that the spectral density of a real random process is an even function.

9. Define casual system.

10. Define transfer function of a system.

PART B — (5 × 16 = 80 marks)

11. (a) (i) The pdf of a random variable X is given by $f(x) = \begin{cases} 2x, 0 \le x \le b \\ 0, \quad \text{otherwise} \end{cases}$.
For what value of b is $f(x)$ a valid pdf? Also find the cdf of the random variable X with the above pdf. (8)

(ii) State and prove memory less property of Geometric distribution. (8)

Or

(b) (i) Find the moment generating function of Poisson distribution and hence find its mean and variance. (8)

(ii) In a normal distribution, 31% of items are under 45 and 8% of items are over 64. Find the mean and the standard deviation of the distribution. (8)

12. (a) (i) The joint probability mass function of (X,Y) is given by $p(x,y) = \frac{1}{72}(2x+3y), x = 0,1,2$ and $y = 1,2,3$. Find all the marginal and conditional probability functions of X and Y. (10)

(ii) The joint pdf of (X,Y) is $f(x,y) = e^{-(x+y)}, x, y \ge 0$. Are X and Y independent? (6)

Or

(b) (i) The joint pdf of a random variable (X,Y) is $f(x,y) = 25e^{-5y}\ 0 < x < 0.2, y > 0$. Find the covariance of X and Y.(8)

(ii) The random variables X and Y each follow exponential distribution with parameter 1 and are independent. Find the pdf of $U = X - Y$. (8)

13. (a) (i) A random process $\{X(t)\}$ is defined by $X(t) = A\cos t + B\sin t, -\infty < t < \infty$ where A and B are independent random variables each of which has a value −2 with probability $\frac{1}{3}$ and a value 1 with probability $\frac{2}{3}$. Show that $\{X(t)\}$ is a wide sense stationary process. (8)

(ii) Suppose the customer arrive at a bank according to Poisson process with mean rate of 3 per minute. Find the probability that during a time interval of two minutes. (1) Exactly four customers arrive. (2) Greater than 4 customers arrive. (3) Fewer than 4 customers arrive. (8)

Or

(b) (i) A man either drives a car or catches a train to go to office each day. He never goes two days in a row by train. But he drives one day , then the next day is just as likely to drive again as he is to travel by train. Now suppose that on the first day of the week, the man tossed a fair dice and drove to work if and only if a 6 appeared. Find the probability that he takes a train on the fourth day and the probability that he drives to work on the fifth day. (8)

(ii) Define semi random telegraph signal process and prove that it is an evolutionary Process. (8)

14. (a) (i) Two random processes $\{X(t)\}$ and $\{Y(t)\}$ are defined as $\{X(t)\} = A\cos(wt+\theta)$ and $\{Y(t)\} = B\sin(wt+\theta)$ where A, B and w are constants and θ is uniformly distributed random variable over $(0, 2\pi)$. Find the cross correlation function of $\{X(t)\}$ and $\{Y(t)\}$. (8)

(ii) The power spectrum of a WSS process $\{X(t)\}$ is given by $S(\omega) = \dfrac{1}{(1+\omega^2)^2}$. Find its auto correlation function $R(\tau)$. (8)

Or

(b) (i) If $Y(t) = X(t+a) - X(t-a)$ prove that $R_{\gamma\gamma}(\tau) = 2R_{XX}(\tau+2a) - R_{XX}(\tau-2a)$ Hence prove that $S_{\gamma\gamma}(w) = 4\sin^2 aw\, S_{XX}(w)$. (10)

(ii) The autocorrelation function of the random telegraph signal process is given by $R(\tau) = a^2 e^{-2\gamma|\tau|}$. Determine the power density spectrum of the random telegraph signal. (6)

15. (a) If $\{X(t)\}$ is a WSS process and if $Y(t) = \int_{-\infty}^{\infty} h(u) \times (t-u)\,du$, prove that :

(i) $R_{XY}(\tau) = R_{XX}(\tau) * h(-\tau)$ and

(ii) $R_{YY}(\tau) = R_{XY}(\tau) * h(\tau)$ where * denotes convolution

(iii) $S_{XY}(\omega) = S_{XX}(\omega)H^*(\omega)$ where $H^*(\omega)$ is the complex conjugate of $H(\omega)$

(iv) $S_{YY}(\omega) = S_{XX}(\omega)|H(\omega)|^2$. (16)

Or

(b) A Random process $X(t)$ is the input to a linear system whose impulse response is $h(t) = 2e^{-t}, t \geq 0$. If the autocorrelation function of the process is $R_{XX}(\tau) = e^{-2|\tau|}$, determine the cross correlation function $R_{XY}(\tau)$ between the input process $X(t)$ and the output process $Y(t)$ X and the cross correlation function $R_{YX}(\tau)$ between the output process $Y(t)$ and the input process $X(t)$. (16)

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