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Question Paper Code : 27333

B.E./B.Tech. DEGREE EXAMINATION, NOVEMBER/DECEMBER 2015

Fourth Semester

Computer Science and Engineering

MA 6453 — PROBABILITY AND QUEUEING THEORY

(Common to Mechanical Engineering (Sandwich) and Information Technology)

(Regulations 2013)

Time : Three hours — Maximum : 100 marks

Answer ALL questions.

PART A — (10 × 2 = 20 marks)

1. A continuous random variable X has the probability density function given by

$$f(x)=\begin{cases}\lambda(1+x^2), & 1\le x\le 5\\ 0, & \text{otherwise}\end{cases}.$$ Find λ and $P(X<4)$.

2. What is meant by memoryless property? Which discrete distribution follows this property?

3. Given the two regression lines $3X+12Y=19, 3Y+9X=46$, find the coefficient of correlation between X and Y.

4. The joint probability density function of bivariate random variable (X,Y) is given by $f(x,y)=\begin{cases}4xy, 0<x<1, & 0<y<1\\ 0 & \text{elsewhere}\end{cases}$. Find $P(X+Y<1)$

5. When is a Markov chain, called homogeneous?

6. Consider a random process $X(t)=\cos(wt+\theta)$, where w is a real constant and θ is a uniform variable in $\left(0,\frac{\pi}{2}\right)$. Show that $X(t)$ is not wide sense stationary.

7. Which queue is called to be the queue with discouragement?

8. What is the effective arrival rate for $(M/M/1):(4/FCFS)$ queueing model?

9. Write down Pollaczek-khintchin formula.

10. What do you mean by bottle neck of a network?

PART B — (5 × 16 = 80 marks)

11. (a) (i) Find the moment generating function of Poisson distribution with parameter λ and hence prove that the mean and variance of the Poisson distribution are equal. (8)

(ii) A component has an exponential time to failure distribution with mean of 10,000 hours.

(1) The component has already been in operation for its mean life. What is the probability that it will fail by 15,000 hours?

(2) At 15,000 hours the component is still in operation. What is the probability that it will operate for another 5000 hours? (8)

Or

(b) (i) The average percentage of marks of candidates in an examination is 42 with a standard deviation of 10. If the minimum mark for pass is 50% and 1000 candidates appear for the examination, how many candidates can be expected to get the pass mark if the marks follow normal distribution? If it is required, that double the number of the candidates should pass, what should be the minimum mark for pass? (8)

(ii) A continuous random variable X has the p.d.f $f(x) = kx^3e^{-x},\ x \geq 0$. Find the rth order moment about the origin, moment generating function, mean and variance of X. (8)

12. (a) Two random variables X and Y have the joint probability density function

$$f(x,y)=\begin{cases} c(4-x-y), & 0\leq x\leq 2, 0\leq y\leq 2 \\ 0, & \text{elsewhere} \end{cases}.$$

Find the equations of two lines of regression. (16)

Or

(b) (i) The joint distribution of X and Y is given by $f(x,y)=\dfrac{x+y}{21}, x=1,2,3, y=1,2.$ Find marginal distributions and conditional distributions. (8)

(ii) If X and Y are independent random variables with probability density functions $e^{-x}, x\geq 0$ and $e^{-y}, y\geq 0$ respectively, find the density function of $U=\dfrac{X}{X+Y}$. (8)

13. (a) (i) Suppose that customers arrive at a bank according to Poisson process with mean rate of 3 per minute. Find the probability that during a time interval of two minutes (1) exactly 4 customers arrive (2) greater than 4 customers arrive (3) fewer than 4 customers arrive.

(8)

(ii) The process $\{X(t)\}$ whose probability distribution under certain condition is given by

$$P[X(t)=n]=\begin{cases}\dfrac{(\mathrm{at})^{n-1}}{(1+at)^{n+1}}, & n=1,2,3,....\\ \dfrac{at}{1+at}, & n=0\end{cases}$$

Show that $\{X(t)\}$ is not stationary. (8)

Or

(b) (i) A man either drives a car (or) catches a train to go to office each day. He never goes two days in a row by train but if he drives one day, then the next day he is just as likely to drive again as he is to travel by train. Now suppose that on the first day of the week, the man tossed a fair die and drove to work if and only if a 6 appeared. Find

(1) the probability that he takes a train on the third day

(2) the probability that he drives to work in the long run (8)

(ii) Show that the random process $\{X(t)=A\cos(w_0t+\theta)\}$ is wide-sense stationary, if A and w_0 are constants and θ is uniformly distributed random variable in (0, 2π). (8)

14. (a) (i) A TV repairman finds that the time spend on his job has an exponential distribution with mean 30 minutes. If he repair sets in the order in which they came in and if the arrival of sets is approximately Poisson with an average rate of 10 per 8 hour day, what is the repairman's expected idle time each day? How many jobs are ahead of average set just brought? (8)

(ii) Patients arrive at a clinic according to Poisson distribution at the rate of 30 patients per hour. The waiting room does not accommodate more than 14 patients. Examination time per patient is exponential with a mean rate of 20 per hour.

(1) Find the effective arrival rate at the clinic.

(2) What is the probability that an arriving patient will not wait?

(3) What is the expected waiting time until a patient is discharged from the clinic? (8)

Or

(b) (i) The engineers have two terminals available to aid their calculations. The average computing job requires 20 minutes of terminal time and each engineer requires some computation one in half an hour. Assume that these are distributed according to an exponential distribution. If the terminals can accommodate only 6 engineers in the waiting space find the expected number of engineers in the computing center. (8)

(ii) Find the system size probabilities for an M/M/C : FIFO /∞ /∞ queueing system under steady state conditions. Also obtain the expression for average number of customers in the system. (8)

15. (a) (i) A repair facility by a large number of machines has two sequential stations with respective rates one per hour and two per hour. The cumulative failure rate of all the machines is 0.5 per hour. Assuming that the system behavior may be approximated by the two-stage tandem (Markovian) queue, determine the average repair time. (6)

(ii) A Laundromat has 5 washing machines. A typical machine breaks down once every 5 days. A repairer takes an average of 2.5 days to repair a machine. Currently, there are three repair workers on duty. The owner has the option of replacing them with a super worker, who can repair a machine in an average of (5/6) day. The salary of the super worker equals the pay of the three repair workers. Break down time and repair time are exponential. Should the Laundromat replace the three repairers with a super worker? (10)

Or

(b) (i) Consider a system of two servers where customers from outside the system arrive at server 1 at a Poisson rate 4 and at server 2 at a Poisson rate 5. The service rates for server 1 and 2 are 8 and 10 respectively. A customer upon completion of service at server 1 is likely to go server 2 or leave the system; whereas a departure from server 2 will go 25 percent of the time to server 1 and will depart the system otherwise. Determine the limiting probabilities, L_s and W_s. (8)

(ii) A one-man barber shop takes exactly 25 minutes to complete one hair-cut. If customers arrive at the barber shop in a Poisson fashion at an average rate of one every 40 minutes, how long on the average a customer in the spends in the shop. Also, find the average time a customer must wait for service? (8)